An Optimal Extraction for Imaging Photometry

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Abstract. I present a new way of extracting the magnitudes of stars from imaging data, with particular emphasis on CCD work. It offers several advantages over previous techniques.
• Better signal-to-noise than software aperture photometry.
• A more reliable estimate of the error than is available for profile fitting
• Freedom from bias due to incorrect estimation of the stellar profile.
For a Fortran implementation see http://www.astro.keele.ac.uk/~timn/

1. The "Optimal" Extraction

A general formula for summing the flux (F) within an aperture is

$$F = \sum_{i,j} W_{i,j}(D_{i,j} - S_{i,j}), \tag{1}$$

where the sum is over all pixels i, j within the aperture. The total counts in a pixel are $D_{i,j}$, and the estimated sky level is $S_{i,j}$. $W_{i,j}$ is the weight for each pixel; for aperture photometry it is 1 within the aperture, and zero outside it.

Clearly the crucial step is deciding what these weights should be to get the optimum signal-to-noise. The steps are as follows.
(1) Fit a model profile to a nearby star (a 2-D Gaussian turns out to be adequate in all cases I've tried so far). The resulting estimated stellar profile is $P^E_{i,j}$, when normalised to one.
(2) As shown by Horne (1986), once you have the normalised estimated profile, if you know the variance for each pixel ($V_{i,j}$), the best signal-to-noise is obtained if the weights in 1 are given by

$$W_{i,j} = \frac{P^E_{i,j}/V_{i,j}}{\sum_{i,j}(P^E_{i,j})^2/V_{i,j}}. \tag{2}$$

We can now substitute equation 2 into equation 1 to obtain the basic formula for optimal extraction. Before doing so, however, we make one further simplifying assumption, that the variance for each pixel is the same. For very faint stars (*i.e.* ones close to the sky limit) this clearly is the case, since the counts in each pixel are mostly due to the sky. This means, though, that the extraction will not be quite optimal for brighter stars. My reasons for making this assumption will become clear in Section 3, but it results in the equation

$$F = \frac{\sum_{i,j} P^E_{i,j}(D_{i,j} - S_{i,j})}{\sum_{i,j} (P^E_{i,j})^2}. \tag{3}$$

2. What if the Model Profile is Wrong?

If the estimated stellar profile is wrong, there will be no systematic bias in the results, provided one is only interested in the relative brightness of one star with respect to another in the same frame. This can be proved if one replaces the flux in equation 1 with the normalised intrinsic profile $P^I_{i,j}$, multiplied by the total flux (T). Then the measured flux is

$$F = T\frac{\sum_{i,j} P^E_{i,j} P^I_{i,j}}{\sum_{i,j} (P^I_{i,j})^2}. \tag{4}$$

The two sums are identical for all stars, and thus an error in estimating the profile has the effect of multiplying all the fluxes by the same (usually small) factor. Obviously this factor is eliminated when the flux from one star is divided by that from another. This explains why the variances are dropped in equation 3, since they would change the sums for bright and faint stars, and thus introduce a systematic error.

3. Comparison with Profile Fitting

A profile fitting program will aim to get the minimum value for χ^2, *i.e.* minimise

$$\chi^2 = \sum_{i,j} (TP^E_{i,j} - (D_{i,j} - S_{i,j}))^2/V_{i,j}. \tag{5}$$

Differentiating with respect to T to find where the minimum χ^2 occurs yields

$$F = \frac{\sum_{i,j} P^E_{i,j}(D_{i,j} - S_{i,j})/V_{i,j}}{\sum_{i,j} (P^E_{i,j})^2/V_{i,j}}. \tag{6}$$

This is very similar to equation 3, except that it includes terms due to the variance. As explained in Section 3, these terms add a magnitude dependent bias to the estimate of the flux, if the observed profile is not accurately measured. Thus there is a danger in profile fitting that there will be systematic errors in the estimated flux that depend on magnitude, if the fitted profile is not a good match to the data.

The error estimates for profile fitting are made using the standard methods for parameter estimation from model fitting with χ^2. These methods rely on the model being a good fit to the data, which is invariably not the case. In contrast, the errors from optimal extraction can be calculated directly using equation 3, and are very robust.

References

Horne K., 1986, PASP, 98, 609