Normalization and the Fermi energy

- We normalize *n(E)* so that $\int_0^\infty n(E)\,dE = N$

Where *N* is the total number of electrons. This will define the constant μ, the chemical potential. So

$$N = \left(\frac{4\pi V}{h^3} (2m)^{3/2} \int_0^\infty \frac{E^{1/2}\,dE}{e^{\beta(E-\mu)}+1} \right)$$

Where at a given temperature T (=1/βk)

$$\mu = \mu(T, V, N)$$

First we can normalize at T=0, which simplifies things. The Fermi Energy is defined as the value of μ at T=0, and it is the maximum electron energy at T=0. Thus we have

$$N = \int_0^{E_F} g(E)\,dE = \frac{8\pi V}{3h^3}(2m)^{3/2} E_F^{3/2}$$

So

$$E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}\frac{N}{V}\right)^{2/3}$$

We define the Fermi Temperature by $E_F = kT_F$

And the equivalent velocity and momentum are

$$V_F = \left(\frac{2E_F}{m}\right)^{1/2}$$

$$p_F = (2mE_F)^{1/2}$$

NB the Fermi Energy depends on the particle mass *m* and the number density. For monovalent metals *V/N* = atomic volume = volume / atom of the solid

Degenerate Fermion Gases

- So far we have taken T=0 and a Fermi gas at T=0 is called ***completely degenerate***, since all the states in each level are full
- At T << Fermi Temperature the distribution is not very different than the T=0 case and the gas is called ***extremely degenerate***. This is the case with all metals at usual temperatures (and also for WD stars - see below)