The general analysis of wave modes in a box (done earlier) is valid. Note that electrons have 2 possible spin states, so the number of modes will be twice that calculated in the general case. Hence the density of states is given by

$$g(\omega)=\frac{V\,\omega^2}{\pi^2 v^3}$$

Where v ('vee') is the electron speed and ω is the angular frequency

Now the waves have $k = v / \omega$, so

$$E=\frac{\omega^2 h^2}{8\pi^2 m v^2}$$

$$\omega^2=\frac{4\pi^2 2 m v^2 E}{h^2}$$

$$\omega=\frac{2\pi(2m)^{1/2} v E^{1/2}}{h}$$

$$\frac{d\omega}{dE}=\frac{\pi(2m)^{1/2} v}{E^{1/2} h}$$

But the number of states in interval dE is g(E)dE = g(ω)dω so we have

$$g(E)=\frac{d\omega}{dE}g(\omega)=\frac{d\omega}{dE}\frac{V}{\pi^2 v^3}\omega^2$$

Substituting we find

$$g(E)=\frac{4\pi V}{h^3}(2m)^{3/2}E^{1/2}$$

Note that $g(E)$ is proportional to V, so we get more states per unit energy in larger boxes

Find the number of particles per unit energy at E, n(E)

- We have $N(E)=f(E)g(E)$ so

$$n(E)=\frac{4\pi V}{h^3}(2m)^{3/2}E^{1/2}\frac{1}{e^{\beta(E-\mu)}+1}$$