Pressure due to a degenerate Fermion gas of electrons

- Kinetic theory for pressure in a perfect gas gave

$$P = \frac{1}{3}\frac{N}{V}\langle m\ v^2\rangle$$

$$P=\frac{1}{3}\frac{N}{V}\langle pv\rangle$$

For a degenerate gas *f(E)=1,* and the number of particles per unit energy at E is

$$n(E)=\left(\frac{4\pi V}{h^3}\right)(2m)^{3/2}E^{1/2}$$

Where 0 <= E <= Fermi energy

But the momentum

$$p=(2mE)^{1/2}$$

$$E=p^2/(2m)$$

$$\frac{dE}{dp}=p/m$$

And the number of particles from E to E+dE is n(E)dE = n(p)dp. So the number of particles per unit momentum p is

$$n(p)=n(E)\frac{dE}{dp}$$

Hence $n(p)=\frac{8\pi V p^2}{h^3}$

Where $0\leqslant p\leqslant p_F$. Now $\langle pv\rangle=\frac{1}{N}\int_0^{p_F} n(p)\,p\,v\,dp$

And pressure $P=\frac{1}{3}\frac{N}{V}\langle pv\rangle$

So electron pressure

$$P_e=\frac{1}{3}\int_0^{p_F}\frac{8\pi p^2}{h^3}\,p\,v\,dp$$

For the ***non-relativistic case*** we have $v = p / m$ and we get

$$P_e=\left(\frac{8\pi}{15mh^3}\right)p_F^5$$

For the ***relativistic case*** $v \sim c$, so

$$P_e=\frac{2\pi c}{3h^3}p_F^4$$

This theory holds for other Fermions as well as electrons