Fermions: Electrons in metals and white dwarf stars

Fermions are particles that obey Fermi-Dirac statistics, and have half-integral spin

The occupation numbers are restricted (at most one particle can be in any state)

Fermions: electrons, positrons, protons, neutrons

Electrons in a box

- Consider a closed box, volume V, in thermal equilibrium, containing a gas of weakly interacting electrons
- These conditions are approximated by `free' conduction electrons in a metal, or electrons in a white dwarf (WD) star
- We must find the energy distribution n(E)=g(E)f(E)

Find the mean number of particles per state, f(E)

- Since electrons are Fermions, we use FD statistics, and have

$$f(E) = f_{FD}(E) = \frac{1}{e^{\beta(E-\mu)}+1}$$

Where at T=0, $\mu = E_F$ (the Fermi energy)

Find the density of states, g(E)

- The solution of Schrödinger's time-independent wave-equation for electron `matter waves' in a box gives solutions which are standing waves with nodes at the walls
- Each possible wavenumber k represents an energy level $E = [h^2/(8\pi^2 m)]k^2$

And a momentum value

$$p = (2mE)^{1/2} = (h/2\pi)k$$

Where m is the electron mass