Mon. Not. R. Astron. Soc. **000**, 000–000 (1997)

On the formation of massive stars

Ian A. Bonnell[1], Matthew R. Bate[2] and Hans Zinnecker[3]

[1] *Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*
[2] *Max-Planck-Institut für Astronomie, Königstuhl 17, D-69117 Heidelberg, Germany*
[3] *Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany*

February 25, 1998

ABSTRACT

We present a model for the formation of massive ($M \gtrsim 10 M_\odot$) stars through accretion-induced collisions in the cores of embedded dense stellar clusters. This model circumvents the problem of accreting onto a star whose luminosity is sufficient to reverse the infall of gas. Instead, the central core of the cluster accretes from the surrounding gas, thereby decreasing its radius until collisions between individual components become significant. These components are, in general, intermediate-mass stars that have formed through accretion onto low-mass protostars. Once a sufficiently massive star has formed to expel the remaining gas, the cluster expands in accordance with this loss of mass, halting further collisions. This process implies a critical stellar density for the formation of massive stars, and a high rate of binaries formed by tidal capture.

Key words: stars: formation – stars: luminosity function, mass function – binaries: general – open clusters and associations: general.

1 INTRODUCTION

Star formation on galactic and extragalactic scales is largely concerned with the formation of massive stars, due to their radiative, kinetic, and chemical feedback into the interstellar medium. However, the formation of massive stars is an, as yet, unsolved problem in astrophysics. Although low and intermediate mass stars ($M \lesssim 10 M_\odot$) are readily explained through gravitational collapse and subsequent accretion (Palla & Stahler 1993), this model fails for high-mass stars. Radiation pressure, from a $\approx 10 M_\odot$ stellar core, on the dust in the infalling gas, halts accretion and thus limits the mass (Yorke & Krugel 1977; Wolfire & Cassinelli 1987; Beech & Mitalas 1994). The aim of this paper is to explore an alternate model that circumvents this problem.

A valuable clue to the formation mechanism of massive stars is found in their environments. Massive stars are rarely found in isolation. Rather, they are predominantly found in the central regions of rich stellar clusters (Zinnecker et al. 1993; Hillenbrand 1997). Furthermore, even the few runaway OB stars are best explained as having originated in the centres of dense systems (Clarke & Pringle 1992).

Accretion in stellar clusters is likely to be an important mechanism in shaping the initial mass function (Zinnecker 1982; Larson 1992). Recent work on the origin of the stellar mass distribution has shown that competitive accretion in clusters naturally results in the formation of the highest mass stars in the deepest parts of the cluster potential, ie, the centre (Bonnell et al. 1997). The alternative, dynamical mass segregation, is unable to account for the location of the massive stars in such young clusters as the ONC (Bonnell & Davies 1997; Hillenbrand & Hartmann 1997). Although this model of competitive accretion cannot apply to very massive stars, where radiation pressure impedes the growth of stellar mass beyond 10 $M_\odot$, it can qualitatively account for the observed trend of the more massive stars to be more centrally located. In the present paper, we combine the model of accretion onto low-mass stars in a protocluster with the effects that such accretion has on the stellar dynamics of the cluster. We show that in this case stellar/protostellar collisions become important in the densest part of the cluster which can lead to the build-up of very massive stars.

In Section 2, we outline why forming massive stars is problematic. In Section 3, we discuss how the core of a cluster evolves during accretion. Section 4 investigates when collisions become important and Section 5 presents the models for the formation of massive stars. The question of binaries among the massive stars is addressed in Section 6. Section 7 explores the implications of this process while our conclusions are presented in Section 8.

2 THE PROBLEM

There are a number of reasons why forming massive stars is problematic within our current understanding of star formation. Firstly, in isolated formation scenarios, there is the problem due to the difficulties of accreting onto massive protostars. The spherical symmetric collapse of massive molecular clouds can be halted once the protostellar core has attained masses $M \gtrsim 10 M_\odot$. This occurs due to the large luminosities of these stars, such that the radiation pressure on dust, which transfers its momentum to the gas, can halt the collapse and reverse the infall (Yorke & Krugel 1977; Wolfire & Cassinelli 1987; Yorke 1993). This limits the amount of mass that can be directly accumulated onto a massive star. For accretion to continue onto a massive protostellar core, the dust properties (abundances and sizes) have to be significantly reduced from those found in the interstellar medium (Wolfire & Cassinelli 1987).

This limit applies to spherically symmetric collapse and accretion. Generally there is some angular momentum in the parent cloud and this results in the formation of a protostellar disc (eg, Yorke, Bodenheimer & Laughlin 1995). Accretion from this disc onto the massive protostar may be able to proceed if the disc is sufficiently thin, if not the disc will be destroyed (Yorke & Welz 1996). Unfortunately, it is presently unclear what disc thickness is necessary or appropriate. Still, to be able to form a massive star this way requires that all the gas collapses to form a disc before the protostar has gained a large fraction of its eventual mass. The problem with this is that collapse is non-homologous, with the central, low angular momentum regions collapsing faster. Plus, gravitational torques will ensure that the disc mass is always less than that of the central object, as they act on a timescale of order the disc's dynamical time, much shorter than the cloud's free-fall time, $t_{\rm ff}$ (Bonnell 1994; Laughlin & Bodenheimer 1994). Thus, a central massive protostar should form while the majority of the cloud is still infalling, and this limits how much mass gathers in the disc and hence the efficiency of the process.

Even if direct collapse could form massive stars, it is difficult to see how this process can account for the massive stars in the centre of the dense clusters such as the Trapezium in the Orion Nebula Cluster [ONC]. This is because the Jeans mass in the Trapezium protocluster cloud must have been small, perhaps as small as 0.3 $M_\odot$(Zinnecker et al. 1993). To see this, we note the Jeans length (fragmentation scale),

$$R_{\rm J} = \left(\frac{5 R_{\rm g} T}{2 G \mu}\right)^{1/2} \left(\frac{4}{3}\pi\rho\right)^{-1/2}, \qquad (1)$$

must have been smaller then the average half separation of the stars that are now present in the cluster center ("touching protostars"), where T is the temperature, ρ is the density, $R_{\rm g}$ is the gas constant, G is the gravitational constant, and μ is the mean molecular weight. From the current stellar number density ($\approx 2-4\times 10^4$ stars pc^{-3} [McCaughrean & Stauffer 1994; Hillenbrand 1997]), we estimate $R_{\rm J} \lesssim 3000$ au.

We can also estimate the mass density ($\approx 100 M_\odot$ now in stars) inside the current core radius ($R_{\rm core} \approx 0.05$ pc, McCaughrean & Stauffer 1994) of the Trapezium, yielding $\approx 2\times 10^5$ $M_\odot$ pc^{-3}, equivalent to an initial smeared out gas density of $\rho \approx 10^{-17}$g cm^{-3} (or more, if the star formation efficiency, SFE, was less than 100 % and some gas was lost). With $R_{\rm J}$ and ρ as above, it follows then that the Jeans mass is small, $M_{\rm J} \lesssim 3 M_\odot$ (assuming 100% SFE), where

$$M_{\rm J} = \left(\frac{5 R_g T}{2 G \mu}\right)^{3/2} \left(\frac{4}{3}\pi\rho\right)^{-1/2}. \qquad (2)$$

This implies a temperature of 90 K. A more realistic temperature of 20 K yields $M_{\rm J} \lesssim 0.3 M_\odot$. To raise the initial Jeans mass to 50 $M_\odot$, the mass of Ori θ^1C, assuming the mass density of the Trapezium cluster core, requires a Jeans length of 8000 au or 0.04 pc (larger than the present day star-star separations) and a temperature of $T \gtrsim 500$ K. These values are clearly implausibly high.

Thus, the stars that form in the centre of dense clusters should have initial masses much smaller than the median mass in the final cluster. Their initial masses are unlikely to be larger than the minimum mass in the cluster. In the ONC, this corresponds to an initial Jeans mass of $M_{\rm J} \approx 0.1 M_\odot$. Furthermore, we know that the massive stars in the centres of young clusters had to form in situ as the clusters are generally not old enough to allow for dynamical mass segregation (Bonnell & Davies 1998). To obtain the observed high masses, subsequent processes including accretion onto the protostars, and, once $M \gtrsim 10 M_\odot$, collisions between the intermediate mass protostars, are required.

The problem of forming a massive star through direct collapse and accretion also applies to scenarios where significant collisional buildup of gaseous clumps occurs before the collapse (eg Bastien 1981; Murray & Lin 1996). Instead, collisions need to occur after the objects have collapsed and attained stellar densities. Random encounters in star forming regions may lead to some protostellar collisions (eg Price & Podsiadlowski 1995) but will in general be very rare. In order for collisions to be significant, a high density environment is required.

3 ACCRETION AND CLUSTER DYNAMICS

The presence of a large amount of gas in a young stellar cluster (e.g. Lada, Strom & Myers 1993) significantly affects the cluster's evolution. This gas, typically comprising 1.5 to 10 times the mass in stars when the cluster is first detectable in the IR, is a remnant of the star formation process, and must have been significantly higher earlier on (ie 100 % gas initially). Accretion of this gas by individual stars increases their masses, and, results in a spectrum of stellar masses even from an initially uniform distribution (Bonnell et al. 1997).

Of direct significance here is the effects accretion has on the cluster dynamics. In the absence of large scale motions such as rotation, the gas velocities will be uncorrelated with those of the stars (especially in the cluster core). Thus, the accreted gas will, in general, have zero net momentum compared to the stars. Now, as each star accretes from the zero momentum gas, it conserves momentum and thus decelerates. This decreases the velocity dispersion in the cluster and hence decreases its kinetic energy. Additionally, the increase in the stellar mass decreases the cluster's potential energy. Thus, the combined effect of accretion in a cluster is to decrease the total energy, and thus make it more bound. This causes the cluster to shrink to compensate for its new, lower, total energy.

For the purposes of this paper, we consider the effects of accretion on the dynamics of the core of a young stellar cluster. The core is modelled as a collection of N stars within a core radius $\mathrm{R_{core}}$, total stellar mass $\mathrm{M_{stars}}$, and total core mass $\mathrm{M_{core}}$, $M_{\rm core} = M_{\rm stars} + M_{\rm gas}$, where $\mathrm{M_{gas}}$ is the mass in gas. The individual components of the core are not modelled. Instead, the core's evolution is followed by considering its global properties and how they evolve under accretion. We consider the core as the central region with a shallow radial density distribution, embedded within a larger, steeper (such as an R^{-2}) distribution. Thus, we assume we can neglect the contribution to the potential and kinetic energies from the surrounding cluster. We also assume that the number of stars in the core is fixed (no stars enter or leave) and that the gas can only enter (and not leave) the core until a massive star is formed.

In the absence of accretion, the core will be in virial equilibrium such that the core radius is

$$R_{\rm core} = \frac{GM_{\rm core}}{v_{\rm disp}^2}, \tag{3}$$

where $\mathrm{v_{disp}}$ is the velocity dispersion in the core. The primary effect of accretion is to perturb the core from this equilibrium by decreasing the total energy. The core then adapts by shrinking, until the velocity dispersion increases sufficiently to reestablish virial equilibrium with a new $\mathrm{R_{core}}$ and $\mathrm{v_{disp}}$. These quantaties can be calculated by first considering how the total energy decreases as the mass is added and secondly by determining what core radius $\mathrm{R_{core}}$ (and $\mathrm{v_{disp}}$) are appropriate for a virialised system at the new, lower, total energy.

In order to calculate how the cluster core shrinks with accretion, we consider the total energy of the stars. The kinetic energy of the stellar component of total mass $\mathrm{M_{stars}}$, is $E_{\rm kin} = 1/2 M_{\rm stars} v_{\rm disp}^2$, and the potential energy of the stars from both the stellar and gaseous components is $E_{\rm pot} = -GM_{\rm stars}(M_{\rm stars} + M_{\rm gas})/R_{\rm core}$. To see the results of accretion more directly, we express the stars' kinetic energy in terms of a characteristic stellar momentum, p, defined from the velocity dispersion as, $p = M_{\rm stars} v_{\rm disp}$. The total energy of the stars, in a cluster core of total mass $\mathrm{M_{core}}$, is then given by:

$$E_{\rm tot} = \frac{p^2}{2M_{\rm stars}} - \frac{GM_{\rm core}M_{\rm stars}}{R_{\rm core}}. \tag{4}$$

From equation 4, we see that adding mass to the core while conserving momentum decreases the total energy. The stars become more bound ($-GM_{\rm core}M_{\rm stars}/R_{\rm core}$) as they have a larger mass. Similarly, their kinetic energy (and velocity dispersion) decreases due to the conservation of momentum, p, ($v_{\rm disp} \propto M_{\rm stars}^{-1}$, $E_{\rm kin} = p^2/2M_{\rm stars} \propto M_{\rm stars}^{-1}$). This new lower total energy, and lower kinetic energy means that the core is now out of virial equilibrium. It then shrinks while conserving energy (increasing the kinetic energy and the $\mathrm{v_{disp}}$) until virial equilibium is restored. Once the cluster core revirialises with total energy $E_{\rm vir}$ (the decreased $E_{\rm tot}$ after accretion), its radius is given by

$$R_{\rm core} = \frac{-GM_{\rm core}M_{\rm stars}}{2E_{\rm vir}}, \tag{5}$$

and the new velocity dispersion is

$$v_{\rm disp} = \sqrt{\frac{GM_{\rm core}}{R_{\rm core}}}. \tag{6}$$

To follow the evolution of the cluster we use equation 4 to calculate the destabilisation of the cluster core due to the accretion, and then equation 5 to calculate the new core radius once it has revirialised. The two remaining unknowns that need to be considered are the timescale on which this happens and the mass of the gas contained in the cluster core. The minimum timescale for any process considering the dynamics of a group of stars is the crossing time, $t_{\rm cross} = R/v_{\rm disp}$, where R is a characteristic radius of the stellar distribution.

The mass of gas ($M_{\rm gas} = M_{\rm core} - M_{\rm stars}$) contained within the core is uncertain and depends on the cluster dynamics and how the gas reacts to the deepening potential. There are three main possibilities. The first is that the accreted gas comes primarily from outside the cluster core, such that the gas mass-fraction in the core is constant. The second is that the accreted gas is completely contained within the core and thus the core mass is constant while the gas mass decreases during accretion. The last possibility is that the gas infall onto the core is greater than the accretion onto the stars and thus the fraction of the core mass in gas increases. Although it is unclear which of these scenarios is correct, we can make a qualitative estimate based the fact that the gas infall time should be comparable to the dynamical, or crossing, time ($t_{\rm cross} = R/v_{\rm disp} = \sqrt{R^3/GM}$). In the near-uniform density core of the cluster, $M \propto R^3$, the crossing time does not change drastically with radius so the gas has time to catch up so the gas can fall in at the same rate as the core's radius decreases. In the surrounding areas with a steeper (R^{-2}) density profile, $M \propto R$, the crossing time increases rapidly with increasing R, and gas that gets left behind cannot catch up to the core. In reality, the core can only shrink with accretion so it will wait for the gas from outlying areas to fall into it, but it does mean that the core is never overwhelmed by gas from outside, it will adapt as quickly as the gas is fed in. We assume that no gas can leave the core; that no dominant outflows capable of removing a significant fraction of the gas occur before the massive stars are formed. Thus, any gas which is initially in the core will remain in the core until accreted. Furthermore, as the accretion-induced contraction of the cluster requires several initial crossing times (see below), significant amounts of gas will fall in from outside the core. Therefore, for the rest of this paper, we generally assume that the gas primarily comes from outside the core, and that the core gas mass is proportional to the stellar mass. We do, however, comment on how the different scenarios affect the results.

The rate at which the cluster core shrinks with added mass depends on how the accretion time, $\mathrm{t_{acc}}$, compares to the crossing time, $\mathrm{t_{cross}}$, as the core will revirialise on the order of its crossing time. For example, if the mass is added on timescales $t_{\rm acc} << t_{\rm cross}$, no matter how much mass is added, the cluster core will revirialise at no less than half its original radius.

3.1 Slow Accretion

For slow mass accretion, $t_{\rm acc} >> t_{\rm cross}$, the core adapts practically instantaneously. In this case, the change in radius as a function of accreted mass, can be derived by differentiating both equations 4 and 5 with respect to mass, remembering that the change in $E_{\rm tot}$ with mass occurs at a constant radius whereas the change in $E_{\rm vir}$ reflects changes in both mass and radius, and that momentum is conserved during accretion. When the core revirialises, the change in $E_{\rm vir}$ and $E_{\rm tot}$ are equal:

$$\left(\frac{\partial E_{\rm tot}}{\partial M_{\rm stars}}\right)_{R_{\rm core}} = \left(\frac{dE_{\rm vir}}{dM_{\rm stars}}\right). \tag{7}$$

Using the fact that $p^2/M_{\rm stars} = GM_{\rm core}M_{\rm stars}/R_{\rm core}$ when virialised, and assuming $M_{\rm core} \propto M_{\rm stars}$, the core radius can be shown to decrease due to accretion as:

$$R_{\rm core} \propto M_{\rm stars}^{-3}. \tag{8}$$

Alternatively, if the core mass is assumed to be constant and the accreted gas is initially fully contained within the cluster core, then $M_{\rm core} = M_{\rm stars} + M_{\rm gas}$ is constant and the cluster shrinks due to the loss of specific kinetic energy. In this case

$$R_{\rm core} \propto M_{\rm stars}^{-2}. \tag{9}$$

In the third scenario where the infall of gas is such that it more than compensates for the accretion, then $M_{\rm core} \propto M_{\rm gas}$, and the cluster core shrinks primarily due to the decrease in potential energy. In this case

$$R_{\rm core} \propto M_{\rm core}^{-1}. \tag{10}$$

The decrease in $R_{\rm core}$ in this case will then be from a combination of the gas infall rate (equation 10) and the stellar mass accretion rate (equation 8).

In all cases, it is clear that even small amounts of accreted mass can significantly decrease the core radius, due to the decrease in total energy (equation 4), and this increases the stellar density. This is important because it dramatically increases the rate of collisions (see §4 below).

3.2 Fast Accretion

For moderate and high mass accretion rates, $t_{\rm acc} \lesssim t_{\rm cross}$, the cluster core does not revirialise immediately, and significant amounts of mass are added before the core can adjust. In order to quantify how this changes the evolution of $R_{\rm core}$, we performed several direct nbody simulations (using a tree-code derived from the SPH/N-body code of Bate, Bonnell & Price 1995) of clusters of 100 stars within a radius of 0.1 pc undergoing constant accretion. These simulations were compared with a toy model of the cluster core's evolution under the influence of accretion. This model calculates, first, how the core shrinks as the total energy decreases under accretion, assuming the mass is added at a constant core radius (equation 4). The amount of mass added is equal to the accretion rate multiplied by the timescale for the process (the revirialisation timescale). Secondly, the cluster then revirialises at a radius corresponding to the new, lower, total energy (equation 5). The timescale for the revirialisation is a free parameter, on the order of a crossing time. By comparing the N-body simulation with this toy model, we determined that the cluster can effectively revirialise every 0.75 $t_{\rm cross}$ (Figure 1). In both cases, the accreted gas is assumed to come from outside the core such that $M_{\rm core} = M_{\rm stars}$ (no unaccreted gas in core). The two cases are well matched after $\approx 1 t_{\rm cross}$, showing that the simple model for the cluster's evolution under mass accretion is a good representation of the dynamics involved. Note that once the cluster has shrunk, the crossing time is much less than the accretion timescale and the variation in cluster radius follows equation 8. It is also worth noting that although the velocity dispersion initially decreases under accretion (due to the conservation of momentum), the subsequent revirialisation will increase the velocity dispersion as potential energy is transferred into kinetic energy (equation 6). For isolated clusters, the comparisons are only valid for approximately a relaxation time (several crossing times for 100 stars), as an isolated cluster of 100 stars starts to dissolve (due to the formation of a hard binary which can absorb the total energy of the cluster). However, if the 100 stars are embedded in a much larger system, it does not dissolve, but simply exchanges members with the surrounding cluster.

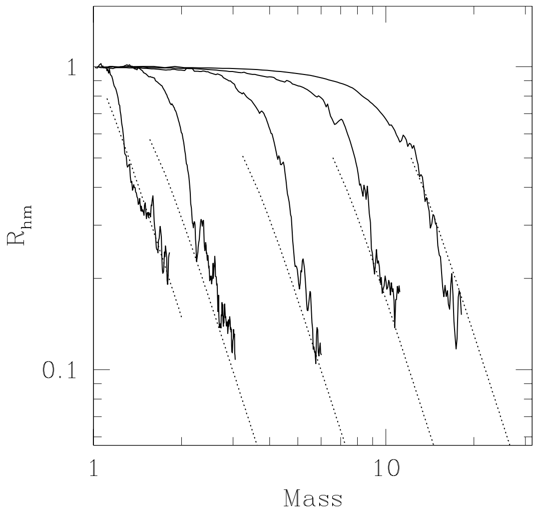

Figure 1. The half-mass radius, in units of the initial half-mass radius, versus total stellar mass, in units of the initial cluster mass, for the nbody simulations (solid lines) and the toy cluster model (dotted lines). The lines are for (from left to right) mean stellar accretion rates of 1×10^{-7}, 5×10^{-7}, 2×10^{-6}, 5×10^{-6} and 1×10^{-5} $M_{\odot}yr^{-1}$.

Thus, for most accretion rates, the rate at which the cluster (and core) shrinks depends not only on the accretion itself but also on the crossing time. This is important as it decreases the differences between the rates of shrinkage (equation 8, 9 and 10) for the various gas dynamics scenarios. The crossing time depends on the amount of mass in the cluster core, such that the crossing time is significantly shorter when all the accreted gas is initially contained within the core as the velocity dispersion is larger. Thus, even though the core does not shrink as much with accreted mass in this case (equation 9), the fact that it can adapt much more quickly balances this difference and the core actually evolves in a similar manner. In the following sections we assume that the accreted gas primarily comes from outside the cluster such that $M_{\rm core} \propto M_{\rm stars}$.

4 TIMESCALES: COLLISIONS VERSUS ACCRETION

In order to ascertain how and when collisions become important, we need to quantify and compare the timescales for collisions and accretion. The accretion timescale, $t_{\rm acc}$, defined as the time it takes to double a star's mass is simply given by

$$t_{\rm acc} = \frac{M_\star}{\dot{M}_\star}, \qquad (11)$$

where $M_\star$ is the star's mass and $\dot{M}_\star$ is the stellar mass accretion rate. The collisional timescale has a more complicated form (see Binney & Tremaine 1987 for a derivation), depending on the stellar velocity dispersion, $v_{\rm disp}$, the stellar density n, and the radius at which mergers occur, $R_\star$:

$$\frac{1}{t_{\rm coll}} = 16\sqrt{\pi} n v_{\rm disp} R_\star^2 \left(1 + \frac{GM_\star}{2v_{\rm disp}^2 R_\star}\right). \qquad (12)$$

The last term incorporates the effects of gravitational focusing in the cluster. Gravitational focussing has an important effect on the collision rate for the cases considered here.

For collisions to be at all relevant, the cluster core must be sufficiently dense. Accretion onto the core of a cluster decreases its radius and increases its density until collisions eventually become significant. Figure 2 illustrates the evolution of the accretion and collisional timescales for a cluster of 100 stars initially contained within a radius of 0.1 pc. The cluster members are all initially 0.1 $M_\odot$ and accrete at a rate of $2 \times 10^{-6} M_\odot {\rm yr}^{-1}$. The collisional timescale is calculated assuming stellar merger radii of 0.1 au and 0.01 au ($R_\star \approx 21.5$ and 2.15 $R_\odot$). We assume that $M_{\rm core} \propto M_{\rm stars}$ and neglect the mass of any gas in the core. The main effects of a non-zero gas mass are to decrease the rate of shrinkage with accretion and to decrease the crossing time. The net effect is little different from Figure 2.

Figure 2 illustrates the relevant timescales for doubling a star's mass ($t_{\rm mass}$). The accretion timescale remains approximately constant at a few $\times 10^5$ years, while the collision timescale decreases from $> 10^8$ years to $< 10^5$ years. Note that the initial cluster crossing time is $t_{\rm cross} = 1.5 \times 10^5$ years and that in both cases, the $t_{\rm coll}$ becomes smaller than $t_{\rm acc}$ in $< 3 t_{\rm cross}$, or $< 4.5 \times 10^5$ years. Thus, collisions become the dominant mode of increasing a star's mass in the cluster core within 5×10^5 years.

The question of what the stellar merger radius should be is not easy to answer as the protostars considered are still in their pre-main-sequence phase, accreting from the surrounding material and their accretion discs. They are in general significantly larger and fluffier than their main sequence counterparts. Pre-main sequence stars, of various masses, less than 10^6 years old, most probably have radii in the range 5 to 20 $R_\odot$ (Palla & Stahler 1993; Beech & Mitalas 1994). Furthermore, the presence of an accreting envelope and disc aids in the capture and merger process (eg Clarke & Pringle 1993; Hall, Clarke & Pringle 1996), thus giving the stars an effectively larger radius. For the remainder of this paper, we assume a merger radius of 0.1 au (21.5 $R_\odot$). It is worth noting that a smaller merger radius still allows for the collisional buildup of massive stars. However, it does imply a larger mean stellar mass at the time when collisions become important as it allows for more accretion.

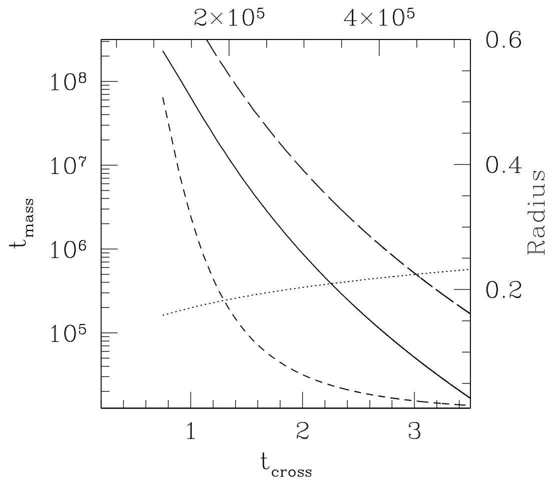

Figure 2. The timescale to double a star's mass either by collisions (solid and long-dashed lines) or through accretion (dotted line) is plotted versus time in units of the initial crossing time, $t_{\rm cross} = 1.5 \times 10^5$ years, (and in years) for a cluster of 100 stars with an initial radius of 0.1 pc. The accretion timescale assumes an accretion rate of 2×10^{-6} $M_\odot {\rm yr}^{-1}$. The merger radius for collisions is taken to be 0.1 (solid line) and 0.01 au (long-dashed line). The cluster radius in units of the initial radius is also plotted (short-dashed line).

5 COLLISIONS AND MASSIVE STARS

In the previous sections, we have seen how the central cores of stellar clusters undergoing accretion eventually shrink sufficiently that collisions dominate the increase in stellar masses. In this section we use the same toy model described above but modified to include collisions and competitive accretion (Zinnecker 1982). Accretion is allowed onto all stars in proportion to their contribution to the gravitational potential, ie

$$\dot{M}_\star = \frac{M_\star^2}{\Sigma_i {M_\star}_i^{\,2}} \dot{M}_{\rm clust}. \qquad (13)$$

This is equivalent to each star accreting with a Bondi-Hoyle type accretion rate (eg Ruffert 1996) assuming that the accreting material is at a uniform density and each star is moving through the medium with the same velocity. The collisions occur on the collisional timescale of each star, $t_{\rm coll}$ (equation 12), and the stars are assumed to merge when they collide. The collisions are assumed to be 100 % efficient in that no mass is lost. In reality, some small amounts of mass will be lost during collisions (eg Davies et al. 1993). Mass loss rates of up to 25 % of the stellar masses were attempted with no significant differences in the results. Accretion is not allowed onto stars more massive than 10 $M_\odot$

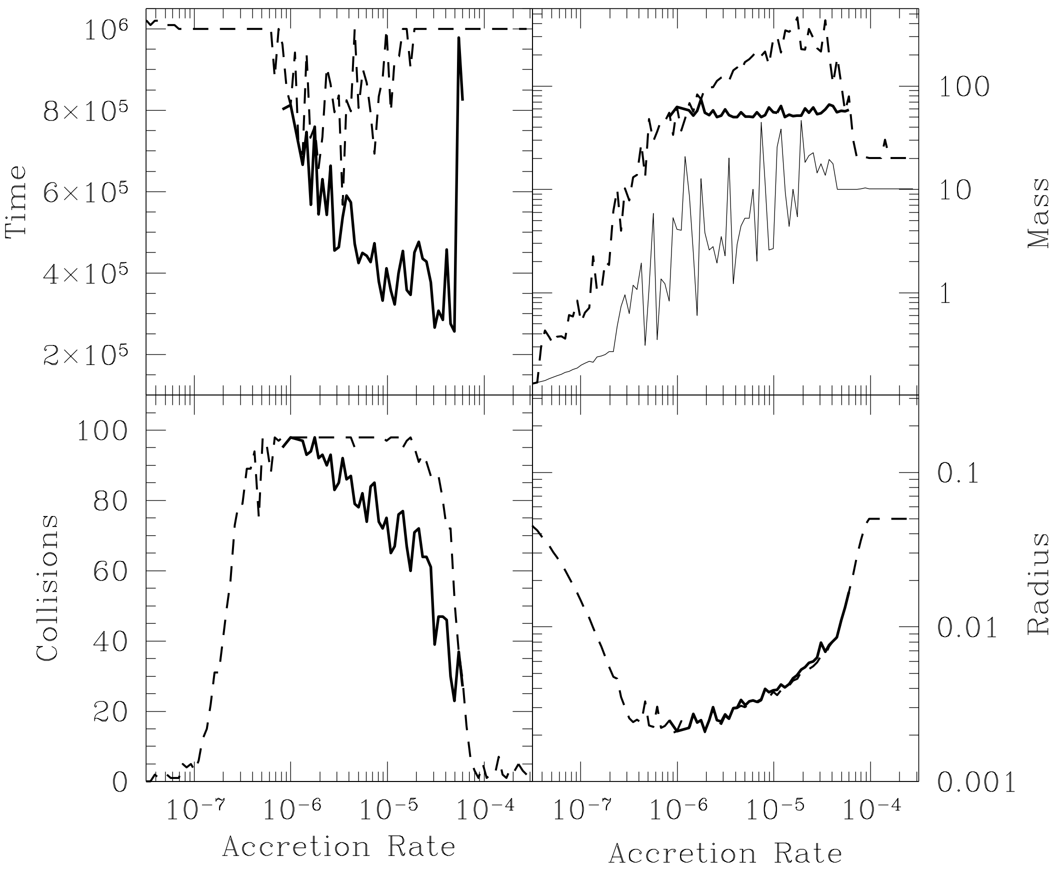

Figure 3. The four panels represent the toy-model calculations of the central core of a cluster containing 100 stars initially within 0.1 pc, evolving under mean stellar accretion rates from 10^{-7}to10^{-4} $M_\odot yr^{-1}$. The stellar merger radius is taken to be 0.1 au. The top left panel shows the time, in years, at the end (dashed line) of the evolution (either 10^6 years or when only 2 stars are left), and the time when a 50 $M_\odot$ star is first formed (heavy solid line). The top-right panel shows the maximum stellar mass (in $M_\odot$) for the same cases plus the median stellar mass at the end of the evolution (light solid line). The bottom left panel shows the number of collisions that occur for the two cases while the radius (in pc) of the cluster core is shown in the bottom right panel.

so that stars more massive than this need to form through collisions. The cluster evolution is followed, allowing for re-virialisation every $0.75t_{\rm cross}$, and recalculating $t_{\rm coll}$ for each star.

The number of stars is only affected by collisions. No stars are allowed to leave or enter the cluster core. In reality some interchange between the core and the surrounding cluster is expected, introducing some lower-mass stars and removing some of the high-mass products. Ejection of stars from the core require an interaction with a hard binary in order to give a kick velocity greater than the velocity dispersion. The finite and, including circumstellar material, relatively large size of the young stars limits such ejections as the interaction will often lead to a merged object (Davies, Benz & Hills 1994). Furthermore, the presence of the gas in the cluster should decrease such ejections and, in general, it would be preferentially the less massive stars that are lost from the core. If many interchanges occur, then the introduction of low-mass stars will increase the number of collisions required to form a massive star. In the extreme case, where many more stars leave the core then enter it, an initially higher number of stars in the core may be necessary. These additional complications are beyond the scope of the present study but should not overtly change the results.

The models are followed for 10^6 years or until only two stars remain (a rare occurrence). Models with varying numbers of stars in the cluster core and varying core radii were investigated. In general, cluster cores with $t_{\rm cross}$ significantly less than 10^6 years (the approximate age at which a complete mass spectrum is observed in the ONC) are the ones in which significant collisional buildup of massive stars occurs.

An illustrative case of a cluster core with 100 stars in 0.1 pc, initial stellar masses of $0.1 M_\odot$ and a stellar merger radius of 0.1 au, evolving under various mean stellar accretion rates is shown in Figure 3. This figure shows the effects of accretion-induced collisions in forming massive stars. The top-left panel shows the period of time required to form a star of 50 $M_\odot$, and the time at which only 2 remain of the original 100 protostars (or 10^6 years). It is worth noting that the evolution requires several $\times 10^5$ to 10^6 years to form a 50 $M_\odot$ star. Thus, massive stars in these clusters are expected to be considerably younger than the mean stellar age, and in some cases, could be up to 10^6 years younger than the

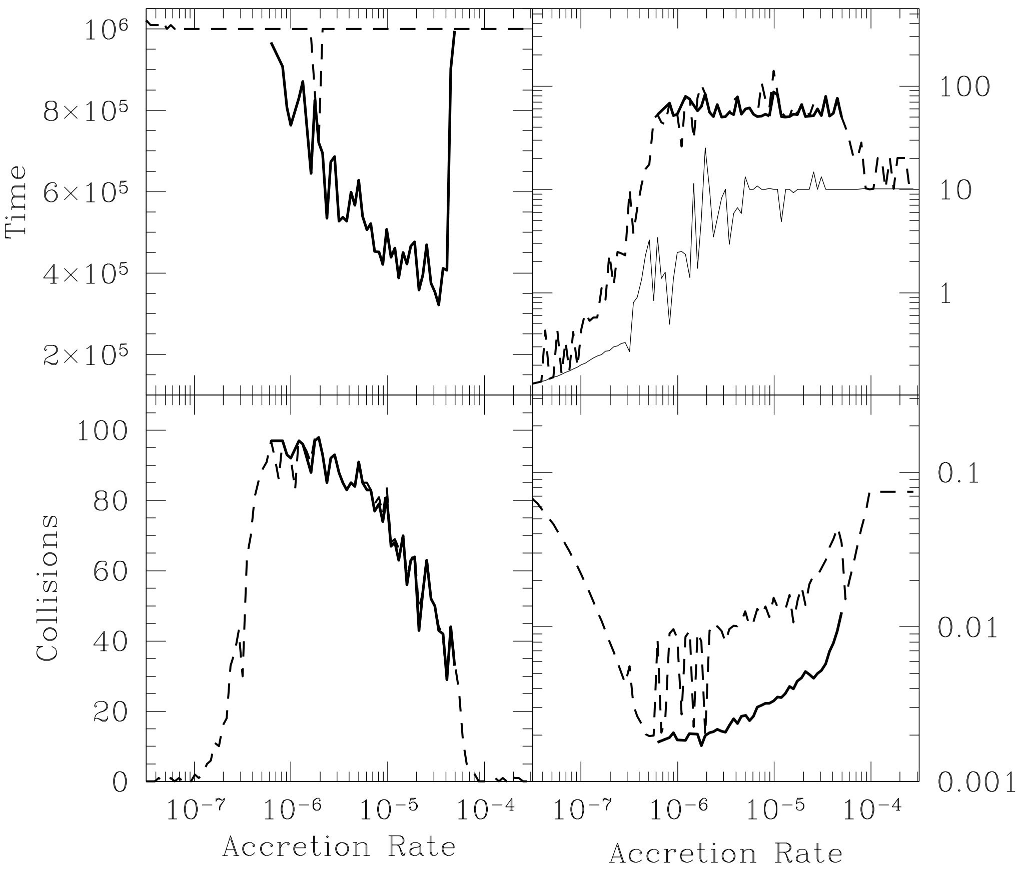

Figure 4. Same as Figure 3 except for a cluster core of 100 stars initially within 0.15 pc and with a gas fraction of 50 % contributing to the gravitational potential. The gas is removed once a star with $M \gtrsim 50 M_\odot$ has formed. The removal occurs over several crossing times.

oldest low-mass stars.

The bottom-left panel of Fig. 3 shows the number of collisions that occur and the bottom-right panel shows the cluster's core radius for the same cases. For small mean stellar accretion rates, few collisions occur as the cluster does not shrink sufficiently within 10^6 years. For large accretion rates, the mass is added very quickly, $t_{\rm acc} << t_{\rm cross}$, and by the time the cluster revirialises, the mean stellar mass is greater than 10 $M_\odot$, halting the accretion. In this case the cluster core does not shrink very much (half the initial core radius) and hence collisions are rare. Maximal shrinkage occurs when $t_{\rm acc} \approx t_{\rm cross} \approx 10^6$ years. Once collisions dominate, the cluster evolves due to the collisions and has no time to shrink further. This occurs at slightly larger radii for larger accretion rates and the mean stellar mass is then larger.

The top-right panel of Fig. 3 shows the maximum mass obtained at either 10^6 years, or when only 2 stars are left, or when a collision has formed a star of mass ≥ 50 $M_\odot$. It is clear from this panel that the maximum mass depends on both the accretion rate and the number of collisions. The maximum mass is greater than 50 $M_\odot$ for accretion rates between 5×10^{-7} and 5×10^{-5} $M_\odot$ yr^{-1}. The median mass at the time that a 50 $M_\odot$ star is first formed is also shown in Fig. 3. The median mass typically lies between a few and 10 $M_\odot$. It is worth noting that for accretion rates $> 10^{-5}$ $M_\odot$ yr^{-1}, accretion alone will produce a 10 $M_\odot$ star in 10^6 years.

As noted above, the exact value of the stellar merger radius does not have a large affect on the results. The above calculations were also performed with a stellar merger radius of 5 $R_\odot$ and gave qualitatively similar results, the main differences being the amount the cluster shrinks as the smaller stellar merger radius necessitates a higher stellar density for collisions, and the mean stellar mass which is larger in the smaller stellar merger radius case.

The maximum mass in Fig. 3 sometimes reaches well in excess of 100 $M_\odot$. This is clearly not physical and is due to the artificial termination points for the evolution (2 stars left, or 10^6 years). A more realistic termination point requires some process that stops the collisions. Up to now, we have ignored the contribution to the potential of the gas being accreted. The gas component in young clusters typically comprises 50 to 90 % of the total mass in the cluster (Lada et al. 1993). The exception to this is the Trapezium where the most massive star, θ^1C, has presumably removed all the gas through its ionising photons and wind. This offers an appealing process of halting further collisions.

In order to include the effect of the gas on the potential, we performed several calculations with either a gas contribution equal to that of the stars or a constant core mass and thus an ever decreasing gas mass. In both cases, the gas is presumed to be removed over several crossing times once a star of sufficient mass (50 $M_\odot$ in this example) is formed.

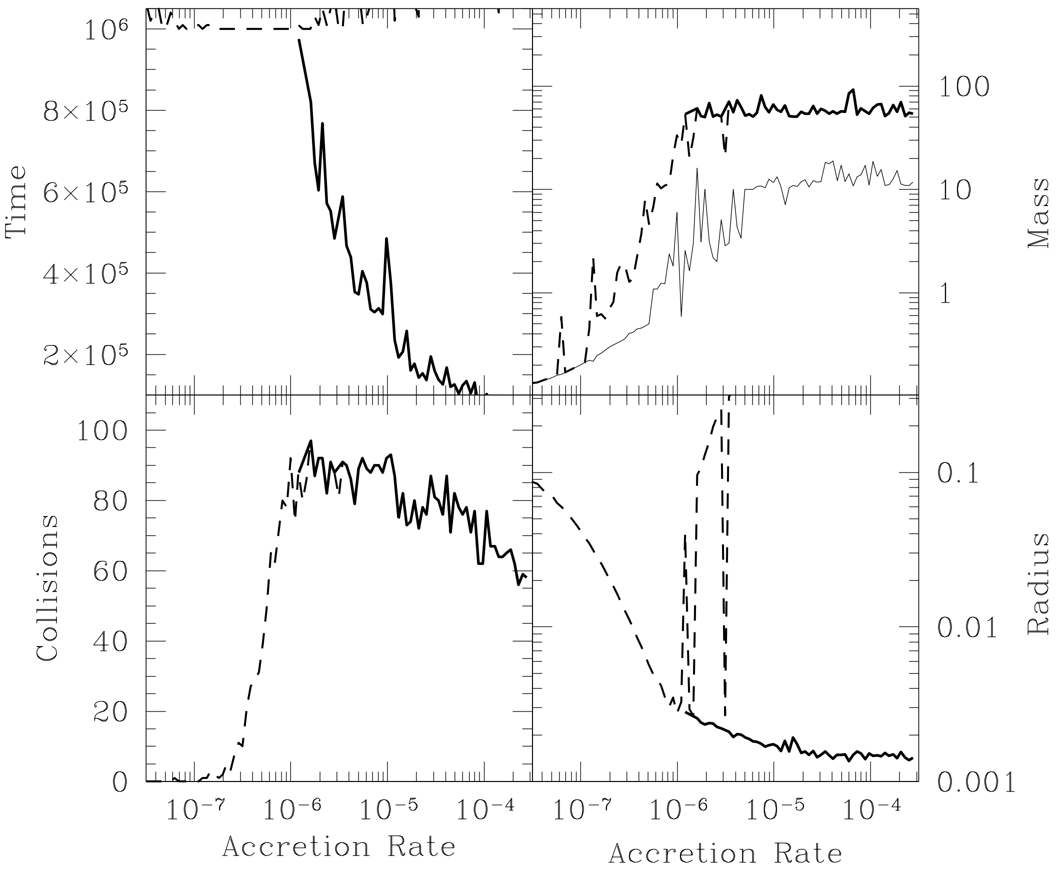

Figure 5. Same as Figure 3 except for a cluster core of 100 stars initially within 0.15 pc and with a constant core mass. The gas is initially contained within the core and has mass equal to that required for the accretion to be sustained for a period of 2×10^6 years. The gas is removed once a star with $M \gtrsim 50 M_\odot$ has formed. The removal occurs over several crossing times.

The gas is expelled due to the ionising photons and wind from the massive star.

Firstly, we consider the effect of the gas when it contributes equally with the stellar mass. Figure 4 shows the evolution in this case for a cluster core of 100 stars initially within a radius of 0.15 pc (the increased radius balances the added mass component of the gas, so that the crossing time is similar). The evolution is very similar to the previous one shown in Figure 3 until a 50 $M_\odot$star is formed. At this point, the gas is removed and the cluster relaxes, increasing its radius, to account for the loss of potential energy. The larger radius, and hence smaller stellar density, effectively eliminates any further collisions. This impedes the formation of the very massive stars ($M > 100 M_\odot$).

Alternatively, if the gas is completely contained within the core initially, and the mass of the core is constant, then the core radius does not shrink as rapidly with accreted mass ($R_{\rm core} \propto M_{\rm stars}^{-2}$), but this is somewhat offset by the much shorter crossing time as the velocity dispersion is higher due to the larger core mass. Such an evolution is illustrated in Figure 5. The initial gas mass is set to twice that required for the desired accretion rate over 10^6 years. Thus, the star formation efficiency increases from near zero to a maximum of 50 %. The minimum cluster radius is well represented by equation 9: $R_{\rm core}/R_{\rm init} = (\dot{M}_\star \times t/M_{\star{\rm init}})^{-2}$. The main difference between the cluster evolution in this case and in the previous ones is that, although the cluster only shrinks as $M_{\rm stars}^{-2}$, due to the large amounts of gas included in the cluster core, the initial crossing time is much shorter. Thus, it can adapt very quickly and there is no maximum value of the mean stellar accretion rate for which significant shrinkage occurs. Any large rate ($\dot{M}_\star \gtrsim 10^{-6}$ $M_\odot {\rm yr}^{-1}$) is sufficient to drive the collisional buildup of massive stars. The large amounts of gas included in the cluster core result in the core being unbound and expanding once the gas is removed by a newly formed massive star ($M_\star \gtrsim 50 M_\odot$). In this case, the massive stars quickly expand into the rest of the cluster.

6 BINARY SYSTEMS

So far, we have neglected binary systems and their possible role in the evolution of the cluster core and in the formation of massive stars. Binary systems are commonly employed as an energy source in globular clusters to stave off core-collapse. The binaries inject energy through binary-single interactions transforming their potential energy into the kinetic energy of the cluster.

In the scenario considered here, the cluster is shrinking due to the accretion. The rate of shrinkage depends on the relation between the accretion timescale and the crossing time. In the extreme case, $t_{\rm acc} >> t_{\rm cross}$, the shrinkage goes as $R_{\rm core} \propto M_{\rm stars}^{-3}$. This is the same rate as is found for a binary system that is accreting zero-angular momentum matter (Smith, Bonnell & Bate 1997). Thus, unless a primordial binary is initially capable of affecting the cluster dynamics, the subsequent accretion and shrinkage should not make it an important energy source for the cluster core.

We do know, however, that many massive stars are in close binary systems (Garmany, Conti & Massey 1980, Mason et al. 1996) so that any theory for their formation should be able to explain why they are in such systems. Their multiplicity can easily be explained in the context of the model presented here where massive star formation is due to collisions. For collisions to be important, many more interactions will occur at slightly greater periastron separations. These interactions can result in the formation of binary systems through tidal capture (eg Mardling 1996). Tidal capture can produce binaries at a few stellar radii but will not form very wide systems. Furthermore, the next interaction can result in a loss of kinetic energy from the binary and thus help the stellar merger rate. In any case, the end result is likely to be of a binary system comprising two of the remaining (proto)stars. The resultant binary is likely to comprise two relatively massive stars, as they both should have accreted a significant amount of mass, and/or undergone collisional mass buildup, before capture. The number of binary systems should be large amongst the massive stars because the tidal capture rate must exceed that for collisions.

Lastly, binary systems among the massive stars can provide a test of this mechanism for the formation of massive stars. The maximum separation of such systems should not, in general, be larger than the hard/soft boundary typical for the stellar density when the collisions occurred. Except for the possibility of binary formation at the very end of the collisional buildup of massive stars, all binaries of separations greater than this critical, density-dependent, separation are broken up by encounters within the core.

7 IMPLICATIONS

There are several implications that can be gleaned from the models presented here. The accretion-induced collisions mechanism requires that the massive stars form in the centre of a large stellar cluster. Thus massive stars should never form in isolation, and those that are found in isolation need to have been ejected from such dense systems.

Accretion-induced collisions can, under reasonable accretion rates, form massive stars in less than 10^6 years. The process does require relatively dense initial conditions, in order to be able to accrete and shrink sufficiently quickly so that collisions are frequent. This is equivalent to requiring $t_{\rm cross} \approx t_{\rm acc} << t_{\rm end}$ where $t_{\rm end} \approx 10^6$ years or some similar maximum time. For initial masses $\approx 0.1 M_\odot$, this implies minimum stellar densities of $\gtrsim 10^4$ stars pc^{-3}. Such densities are typical only of the central regions of large clusters such as the Orion Nebula cluster, where these massive stars are indeed found (McCaughrean & Stauffer 1994; Hillenbrand 1997).

The time required for accretion-induced collisions to form a massive star may imply a considerable age difference between the older low-mass stars and the younger high-mass stars. This age difference is at least several $\times 10^5$ years and can be as much as 10^6 years, depending on the initial cluster density and the accretion rate. Thus stars such as θ^1C in the ONC may have only recently "turned on" and started to affect the gas in the cluster and in discs around the low-mass stars. The Trapezium is then the remnant of the central dense core where the collisions occurred and which has recently expanded to its present configuration due to the gas expulsion.

The collisional buildup of massive stars also has implications for the intermediate-mass stars. It is these stars that, having attained their mass through accretion in the cluster core, are required to collide to form the massive stars. Thus, the formation of the massive stars implies a removal of intermediate mass stars. Systems with massive stars could then demonstrate a relative lack of intermediate-mass stars. There is a tantalising hint of the relative paucity of intermediate-mass stars in the ONC (Hillenbrand 1997) and in a few other clusters (Wilner & Lada 1991) although it could be due simply to statistical fluctuations.

Finally, since, in this model, massive ($m \gtrsim 10 M_\odot$) and intermediate mass ($m \lesssim 10 M_\odot$) stars are formed by different mechanisms, the resultant mass distribution may display some structure at $\approx 10 M_\odot$ (not a single power-law IMF).

8 CONCLUSIONS

Accretion-induced collisions in the core of a dense, young stellar cluster can form massive stars. This circumvents the problem of accreting directly onto stars with masses $> 10 M_\odot$ where radiation pressure can halt the infall. Gas accretion onto individual members of the core of a stellar cluster forces the core to shrink. If the accretion timescale is comparable to the initial crossing time, the core can shrink sufficiently that collisions become significant. The collisions involve intermediate mass stars that are formed by accretion onto initially lower mass protostars. The actual stellar merger radius does not overtly affect this process as the cluster core will shrink due to the accretion until collisions become dominant. The collisions are most probably halted by the ejection of the gas contained in the core once a sufficiently massive star is formed ($\approx 50 M_\odot$).

This mechanism for forming massive stars implies that they need to form in the centre of rich, dense young stellar clusters. They could appear significantly younger than the mean age of the low-mass stars. There is also the potential that the collisions will deplete the number of intermediate mass stars as it is they that will collide to form the massive stars. Close binary systems should be common amongst the massive stars, formed through tidal capture of stars which have close interactions but do not collide.

ACKNOWLEDGMENTS

We thank Cathie Clarke, Jim Pringle, Melvyn Davies, Hal Yorke and Russell Cannon for many insightful discussions. We also thank an anonymous referee whose suggestions helped significantly improve the text. IAB acknowledges support from a PPARC advanced fellowship.

REFERENCES

Bastien P., 1981, A&A, 93 160
Bate M.R., Bonnell I.A., Price N.M, 1995, MNRAS, 277, 362

Beech M., Mitalas R., 1994, ApJS, 95, 517
Binney J., Tremaine S., 1987, in Galactic Dynamics, Princeton University.
Bonnell I. A., 1994, MNRAS, 269, 837
Bonnell I. A., Bate M. R., Clarke C. J., Pringle J. E, 1997, MNRAS, 285, 201
Bonnell I. A., Davies M. B., 1997, MNRAS, in press
Clarke C. J., Pringle J. E., 1992, MNRAS, 255, 432
Clarke C. J., Pringle J. E., 1993, MNRAS, 261, 190
Davies M.B., Benz W., Hills J.G., 1994, ApJ, 424, 870
Davies M. B., Ruffert M., Benz W., Müller E., 1993, A&A, 272, 430
Garmany C. D., Conti P. S, Massey P., 1980, ApJ, 242, 1063
Hall, S. M., Clarke C. J., Pringle J, E., 1996, MNRAS, 278, 303
Hillenbrand L. A., 1997, AJ, 113, 1733
Hillenbrand L. A., Hartmann L., 1997, preprint
Lada E. A., Strom K. M., Myers P. C., 1993, in *Protostars and Planets III*, eds E. Levy and J. Lunine, University of Arizona, p. 245
Larson R. B., 1992, MNRAS, 256, 641
Laughlin G., Bodenheimer P., 1994, ApJ, 436, 335
Mardling R. A., 1996, in *Evolutionary Processes in Binary Stars*, eds R. A. M. J. Wijers, M. B. Davies and C. A. Tout, Kluwer, Dordrecht, p. 81
Mason B. D., Hartkopf W. I., Gies D. R., McAlister H. A., Bagnuolo W. G., 1996, in *The Origins, Evolution and Destinies of Binary Stars in Clusters*, eds E. F. Milone and J. C. Mermilliod, ASP, p. 40
McCaughrean M. J., Stauffer J, R., 1994, AJ, 108, 1382
Murray S. M., Lin D. N., 1996, ApJ, 467, 728
Palla F., Stahler S. W., 1993, ApJ, 418, 414
Price N. M., Podsiadlowski Ph., 1995, MNRAS, 271 1041
Ruffert M., 1996, A&A, 311, 817
Smith K. W., Bonnell I. A., Bate M. R., 1997, MNRAS, 288, 1041
Wilner D. J., Lada C. J., 1991, AJ, 102, 1050
Wolfire M. G., Cassinelli J. P., 1987, ApJ, 319, 850
Yorke H. W. 1993, in *Massive Stars: Their Lives in the interstellar Medium*, eds. J. Cassinelli, E. Churchwell, ASP Conf. Ser. 35:45–55.
Yorke H. W., Bodenheimer P., Laughlin G., 1995, ApJ, 443, 199
Yorke H. W., Krügel E., 1977, A&A, 54, 183
Yorke H. W., Welz A., 1996, A&A, 315, 555
Zinnecker H., 1982 in *Symposium on the Orion Nebula to Honour Henry Draper*, eds A. E. Glassgold et al., New York Academy of Sciences, p. 226
Zinnecker H., McCaughrean M. J., Wilking B. A., 1993 in *Protostars and Planets III*, eds E. Levy and J. Lunine, University of Arizona, p. 429