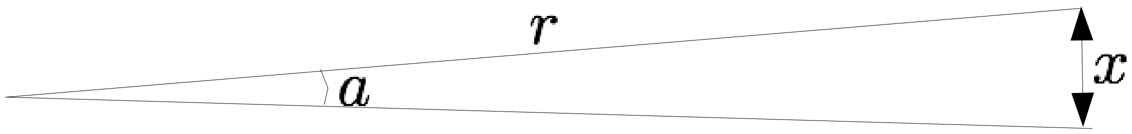
Size, distance and angle



Astronomers use geometry to measure the sizes and distances of planets and nearby stars. If we can draw a triangle like the one above then there's a geometric relationship between size x, distance r and angle a:

$$\boxed{x = r \times a \times \frac{\pi}{180}}$$

This relationship is true for small values of the angle *a* measured in degrees. For angles of less than 30 degrees it gives errors of less than 10%) *

Equipment for tasks:
large protractor
long pieces of string
metre rule and/or measuring tape

Task 1. Checking the relationship.

We can check the relationship by looking at some objects where we can measure all three quantities: size, distance and angle. Find some medium-size objects (such as desks, radiators or pictures) and measure the angle from several metres away by holding the middle of the string and using the string to make straight lines to the edges of the object. Use a protractor to measure the **angle a** between the strings, and the **size x** and **distance r** of the object using a metre rule or measuring tape. Put the numbers in a table like this one:

Angle *a* (degrees)	Distance *r* (m)	Size *x* (m)	$\frac{a \times r}{x}$
11	6.80	1.34	

Fill in the last column by calculating angle x distance / size. You should find that it's about

$$\frac{a \times r}{x} = \frac{180}{\pi} \simeq 57$$

which is another way of rearranging the equation at the top.
Why aren't all your results exactly 57?

* Note for the mathematically minded: this equation is a special case of $x = 2 \times r \times \tan(a/2)$
When the angle *a* is small , we can approximate $\tan(a/2) \simeq a/2$ where a is measured in radians (1 radian = $180/\pi$ degrees).

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Now that we know the equation works, we can use it to calculate things we couldn't measure directly.

Task 2. Sizes.

If we can't measure the size of something directly, we can calculate the size from its distance and angular size.

To try this out, measure the **distance *r*** to some large objects (such as a building, tree or person at the other end of the classroom), and measure their **angular size *a*** using a protractor. You might need to sight along a pencil or metre rule to get a good measurement of the angle. (As a rough estimate, a fist held at arm's length spans 10 degrees.)

Then calculate the **size *x*** using $x = \frac{r \times a}{57}$

Angle *a* (degrees)	Distance *r* (m)	Size *x* (m)
10	32	5.6

We can use this for much more distant objects, if we know their distances. What physical size does an angular size of half a degree correspond to at the following distances?

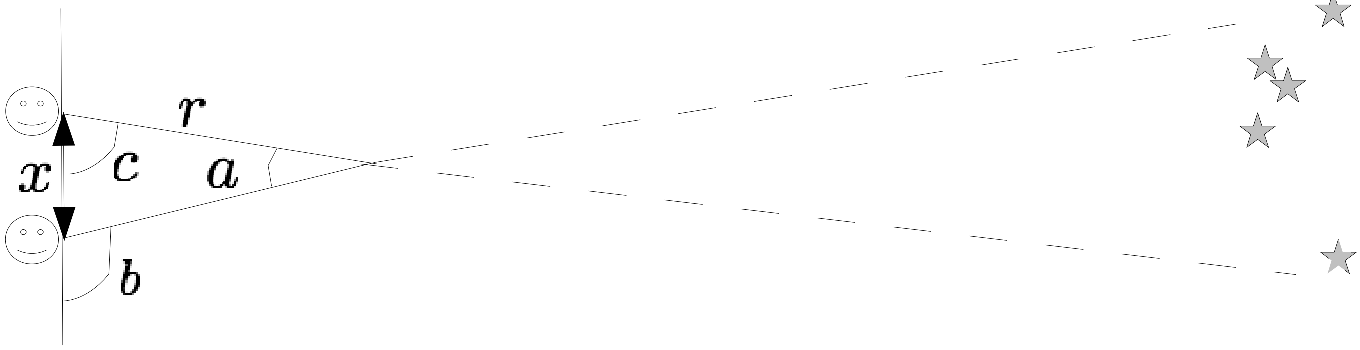
Angle *a* (degrees)	Distance *r* (m)	Size *x* (m)
0.5	1km =	
0.5	Distance to Moon =	
0.5	Distance to Sun =	

Compare these sizes to the measured sizes of the Moon and Sun. How close are they? What angle would you have to use to reproduce the true sizes of the Moon and Sun? Is half a degree a good estimate of the angular size?

Task 3. Parallax.

As we move, the angle at which we see an object at changes. The closer the object, the more the angle changes.

We are still calculating using a triangle with distance and angle but instead of size we have a measurement baseline, which is the distance between our two measurement positions.



The same triangle formula applies for small angles *a* (measured in degrees). Rearranged to give us the distance:

$$r = \frac{\pi}{180} \times \frac{x}{a}$$

To calculate the angle a without going to our distant object, measure how much the viewing angle changes between the two observing positions. Each angle measurement should use the same reference (eg. North if using a compass, or a straight line between the two). On the diagram above, $a = b - c$

Measure some distances using parallax.

b (deg)	*c* (deg)	*a* (deg)	x (m)	r (m)

When we measure the change in angle relative to background stars,this method is called **parallax.** We pick background stars which are so far away that their apparent angle doesn't change, using the Earth's orbit as a baseline.

Parallax is the main method of estimating distances to nearby stars, and is used by the Hipparcos and Gaia satellites

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