PHY 3145 Topics in Theoretical Physics
Astrophysical radiation processes - Dr J. Hatchell

Special Relativity - summary

1 Lorentz transform (LT)

With 4-vectors formulated as (a_x, a_y, a_z, a_t) (see below) and frame S' moving at velocity v in the x-direction with respect to frame S,

From observer's/lab frame S to moving frame S'

$$\begin{aligned} a'_x &= \gamma(a_x - \beta a_t) \\ a'_y &= a_y \\ a'_z &= a_z \\ a'_t &= \gamma(a_t - \beta a_x) \end{aligned}$$

where $\beta = v/c$ and

$$\gamma = \frac{1}{\sqrt{(1 - v^2/c^2)}}.$$

We will often use the reverse transformation to relate quantities in a particle's rest frame S' to the lab or observer's frame S, in which the particle moves at velocity v in the x-direction.

From moving frame S' to observer's frame S:

$$\begin{aligned} a_x &= \gamma(a'_x + \beta a'_t) \\ a_y &= a'_y \\ a_z &= a'_z \\ a_t &= \gamma(a'_t + \beta a'_x) \end{aligned}$$

2 Transformation of E and B

From lab/observer's frame to moving frame. In reverse, the velocity switches sign.

Magnetic field

$$\begin{aligned} B'_x &= B_x \\ B'_y &= \gamma(B_y + \tfrac{v}{c^2} E_z) \\ B'_z &= \gamma(B_z - \tfrac{v}{c^2} E_y) \end{aligned}$$

Electric field

$$\begin{aligned} E'_x &= E_x \\ E'_y &= \gamma(E_y - vB_z) \\ E'_z &= \gamma(E_z + vB_y) \end{aligned}$$

3 4-vectors

space-time	$\mathbf{X} = (x, y, z, ct) = (\mathbf{x}, ct)$
4-velocity	$\mathbf{U} = \dfrac{d\mathbf{X}}{d\tau} = (\gamma_u \mathbf{u}, \gamma_u c)$

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\begin{array}{rrcl}
\text{4-momentum} & \mathbf{P} & = & m_0\mathbf{U} = (\gamma_u m\mathbf{u}, \gamma_u m_0 c) = (\mathbf{p}, \gamma_u mc) \\
\text{for photon} & \mathbf{K} & = & (\mathbf{k}, \omega/\mathbf{c}) \\
\text{4-acceleration} & \mathbf{A} & = & \frac{d\mathbf{U}}{d\tau} = (\gamma_u^2\mathbf{a} + \gamma_u\frac{d\gamma_u}{dt}\mathbf{u}, c\gamma_u\frac{d\gamma_u}{dt}) = (\gamma^2\mathbf{a} + \frac{\gamma^4}{c^2}(\mathbf{a}\cdot\mathbf{v})\mathbf{v}, \frac{\gamma^4}{c}\mathbf{a}\cdot\mathbf{v}) \\
\text{4-force} & \mathbf{F} & = & m_0\mathbf{A} \text{ or } \frac{d\mathbf{P}}{d\tau} = \gamma_u\frac{d}{dt}(\mathbf{p}, \gamma_u m_0 c) = (\gamma_u\mathbf{f}, \frac{\gamma_u}{c}\mathbf{f}\cdot\mathbf{u})
\end{array}
$$

where m_0 is the rest mass and τ is the proper time (time in the rest frame of the particle). For these quantities will try to use capital letters to differentiate 4-vectors $\mathbf{A}$ from 3-vectors $\mathbf{a}$.

If $\mathbf{A}$ and $\mathbf{B}$ are 4-vectors then $\mathbf{A}\cdot\mathbf{B}$ is **invariant** under the Lorentz transform. Therefore $\mathbf{A}\cdot\mathbf{A}$ (also known as the norm) is Lorentz invariant.