PHY 3145 Topics in Theoretical Physics

Astrophysical Radiation Processes - Dr J. Hatchell

Homework questions

Grading: * = easy/quick, ** = medium, *** = hard/long

1 Radiative Transfer

1. * For interstellar dust the absorption coefficient is usually written as $\alpha = \rho\kappa$ where κ is the opacity per unit mass, and ρ is the density. Show that in the optically thin case the emitted intensity is proportional to the column density (mass per unit area), whereas in the optically thick case it only depends on temperature (and frequency).

2. ** For line emission, the absorption and emission coefficients can be written in terms of the Einstein coefficients for spontaneous emission A_{21}, absorption B_{12} and stimulated emission B_{21} as
$$j_\nu = h\nu_{21} n_2 A_{21} \qquad \alpha_\nu = h\nu_{21}(n_1 B_{12} - n_2 B_{21}).$$
Given the first Einstein relation $g_2 B_{21} = g_1 B_{12}$ and the level populations n_1 and n_2 are in thermal equilibrium, use Kirchoff's law to find the second Einstein relation between A_{21} and B_{21}.

2 Radiation from accelerated charges

1. * **Gyrotron radiation.** A charge q moving at (non-relativistic) velocity $\mathbf{v}$ is accelerated in a magnetic field $\mathbf{B}$ at an angle θ to the direction of motion. Show that the total power radiated is
$$P = \frac{e^2}{6\pi\epsilon_0 c^3}\left(\frac{evB\sin\theta}{m}\right)^2.$$

2. * Write down the power emitted per unit solid angle by an accelerated charge. Show that the integral over solid angle returns Larmor's formula (Hint: $d\Omega = \sin\theta d\theta d\phi$).

3. ** **Gyrotron radiation.** A charge q circles in a magnetic field of strength B. What is the angular frequency of the motion? By considering a stationary observer's view of the angular power pattern of the circling charge, show that the emission is all at a single frequency, the gyrofrequency $\omega_g = qB/m$.

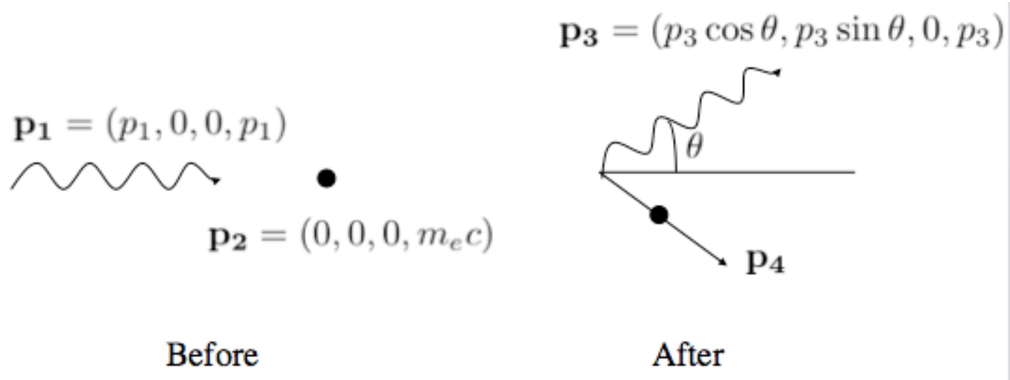
3 Relativistic modifications

1. ** **Relativistic bremsstrahlung.** An electron travelling at relativistic velocity v in the x-direction passes an ion of charge Ze at impact parameter b at time $t = 0$ and is accelerated by the Coulomb interaction. The electric field at the position of the electron is given by
$$\mathbf{E} = \left(\frac{Zevt}{4\pi\epsilon_0[b^2 + (vt)^2]^{3/2}}, \frac{Zeb}{4\pi\epsilon_0[b^2 + (vt)^2]^{3/2}}, 0\right)$$
Lorentz transform the field to the electron's rest frame, and the time t to proper time t', and hence calculate the rest-frame acceleration.

2. ** **Superluminal motion** Plot a graph of the apparent transverse velocity of a superluminal jet as a function of the angle to the line of sight α. Where is the apparent velocity equal to zero? What happens to the angle at which the apparent velocity is a maximum as v approaches c?

3. ** **Relativistic beaming** Sketch the angular power pattern emitted by a charge with acceleration $\mathbf{a}$ *(a)* in θ where θ is the angle between the acceleration and the emission; *(b)* in ϕ where ϕ is the angle in a plane perpendicular to the acceleration. The maximum power per unit solid angle emitted in the charge rest frame is P. If the charge moves with relativistic velocity $v = 0.9\ c$ perpendicular to the acceleration, at what angle from the direction of motion would an observer see *(i)* no radiation? *(ii)* maximum radiation?

4. *** **Compton scattering.** A photon scatters from an electron with 4-momenta as shown in the diagram below. By considering $\mathbf{p_4} \cdot \mathbf{p_4}$ together with conservation of momentum, show that the wavelength shift is
$$\lambda_3 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta).$$
[Hint: for photons the norm $\mathbf{p_1} \cdot \mathbf{p_1} = \mathbf{p_3} \cdot \mathbf{p_3} = 0$ and for electrons $\mathbf{p_2} \cdot \mathbf{p_2} = \mathbf{p_4} \cdot \mathbf{p_4} = m_o^2 c^2$.]



5. *** By splitting into components perpendicular and parallel to the velocity, show that the instantaneous rest frame acceleration a' and lab. frame acceleration are related by $|a|'^2 = \gamma^4(\gamma^2|a_{\|}|^2 + |a_{\perp}|^2)$ and therefore rewrite the Larmor formula in terms of the lab. frame acceleration.

4 Radiation in the real Universe

1. * Look on the Web for further examples of the radiation processes discussed in the lectures. For example:
   - Diamond Light Source `http://www.diamond.ac.uk`, eg. their YouTube video "Synchrotron Technology"
   - Jupiter synchrotron, eg. Bolton et al. 2002, Nature, 415, p.987
   - Radio jets, eg. `http://apod.nasa.gov/apod/ap041211.html`

2. *** The electrons in a cathode ray tube are accelerated by an electric field $E = V/d$ where $V = 2000\ V$ and $d = 2$ cm. How much power is radiated by one electron during the acceleration? If the acceleration is from zero velocity over distance d, what is the final velocity of the electrons and the acceleration time? What can you say about the frequency spectrum? The electron gun in a television accelerates electrons towards the screen. In what direction is the radiation a minimum?