PHY 3145 Topics in Theoretical Physics
Astrophysical Radiation Processes - Dr J. Hatchell

Answers to homework questions

1 Radiative Transfer

1. *Optically thin* $\tau = \rho\kappa S \ll 1$ *so* $I = B_\nu(\nu, T)\kappa N$ *where* $N = \rho S$ *is the column density or mass per unit area along the line of sight.*

   Optically thick $\tau \geq 1$ *so* $I = B_\nu(\nu, T)$ *- for optically thick thermal emission, emission is just the black-body function.*

2. *Hint: in thermal equilibrium, level populations follow the Maxwell-Boltzmann distribution* $\frac{n_2}{g_2} = \frac{n_1}{g_1}\exp(-\frac{h\nu}{kT})$ *and* $j_\nu/\alpha_\nu = B_\nu(T, \nu)$, *so* $A_{21} = \frac{2h\nu^3}{c^2}B_{21}$

2 Radiation from accelerated charges

1. *Hint: use* $a = \frac{qvB\sin\theta}{m}$
2. *Another hint:* $\int \sin^3\theta\, d\theta = \int(1 - \cos^2\theta)\sin\theta\, d\theta$
3. *Power varies as* $P(t) \propto (\sin\theta)^2 = (\sin(\omega_g t))^2$ *so electric/magnetic field vary as* $\sin(\omega_g t)$ *which is an EM wave of frequency* ω_g.

3 Relativistic modifications

1. *See handouts under "Acceleration of electron and total power".*

$$a = \left(\frac{Ze^2}{4\pi\epsilon_0 m}\right)\left[\frac{(\gamma vt')^2}{[(\gamma vt')^2 + b^2]^3} + \frac{(\gamma b)^2}{[(\gamma vt')^2 + b^2]^3}\right]^{1/2}$$

2. *Plot* $v_{\rm app} = (v/c)\sin(\theta)/(1 - (v/c)\cos(\theta)$ *for various* $(v/c) < 1$. *Apparent velocity is zero when jet is directed towards or away from observer. The angle for maximum velocity reduces as* v *increases towards* c.
3. *(a) figure-of-eight,* $\propto \sin^2\theta$*; (b) no variation with* ϕ*, circular power pattern. (i) Use aberration formula. Minimum at* 25 deg*; (ii) maximum at* 0 deg.
4. *Handwritten Compton scattering solution handed out. See also R&L 7.1 and Longair 4.3.2.*
5. *Hint: square*

$$\mathbf{A} = (\gamma^2\mathbf{a} + \frac{\gamma^4}{\mathbf{c}^2}(\mathbf{a}\dot{\mathbf{v}})\mathbf{v}, \frac{\gamma^4}{\mathbf{c}}\mathbf{a}\dot{\mathbf{v}})$$

   in lab. frame and comoving frame noting that $|\mathbf{v}'| = 0$ *in comoving frame.*

4 Radiation in the real Universe

1. * *Web question - no answer required*
2. *** *Use* $F = ma = eE = eV/d$. *Power radiated* 1.8×10^{-21} *W. Final velocity is* $0.1c$, *in* $1.5 \times 10^{-}9s$. *Frequency spectrum 700 MHz plus shorter harmonics (from pulse length). The radiation is a minimum parallel to the acceleration ie. for viewers facing the screen.*