PHY3145 Topics in Theoretical Physics

Astrophysical Radiation Processes

5:Synchrotron and Bremsstrahlung spectra

Dr. J. Hatchell, Physics 406, J.Hatchell@exeter.ac.uk

Course structure

1. **Radiation basics**. Radiative transfer.
2. **Accelerated charges produce radiation**. Larmor formula. Acceleration in electric and magnetic fields – non-relativistic bremsstrahlung and gyrotron radiation.
3. **Relativistic modifications I**. Doppler shift and photon momentum. Thomson, Compton and inverse Compton scattering.
4. **Relativisitic modifications II**. Emission and arrival times. Superluminal motion and relativistic beaming. Gyrotron, cyclotron and synchrotron beaming. Acceleration in particle rest frame.
5. **Bremsstrahlung and synchrotron spectra.**

Bremsstrahlung spectrum: Plan of Action

The spectral energy distribution for Bremsstrahlung is built up in several stages.

i. **Single electron-ion encounter**. The single-encounter spectrum P(ω) is derived from the Fourier transform of a(t) where a is the acceleration in the Coulomb field.

ii. **Multiple interactions**

Multiple *ions* leads to an integration over the range of impact parameters *b*

Multiple *electrons* require an integration over the electron velocity distribution. For thermal bremsstrahlung this is a Maxwellian distribution.

The results are the emission and absorption coefficients per unit volume.

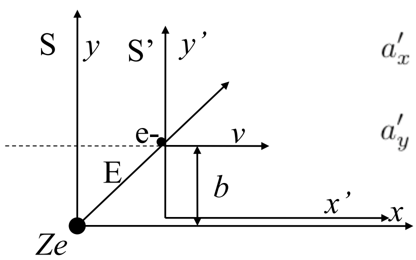
iii. **Thermal and non-thermal spectra**

The equation of radiative transfer gives us the observed spectrum from the emission and absorption coefficients

For full derivation see Longair ch. 2&3, R&L ch. 5.

Acceleration of electron and total power

Bremsstrahlung = **free-free radiation** = radiation by an unbound charged particle due to acceleration by another charged particle



$$a'_x = \frac{eE'_x}{m_e} = \frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{\gamma v t'}{[(\gamma v t')^2 + b^2]^{3/2}}$$

$$a'_y = \frac{eE'_y}{m_e} = \frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{\gamma b}{[(\gamma v t')^2 + b^2]^{3/2}}$$

Relativistic accelerations

$$P' = \frac{dW'}{dt'} = \frac{e^2}{6\pi\epsilon_0 c^3}\left(|a'_x(t')|^2 + |a'_y(t')|^2\right)$$

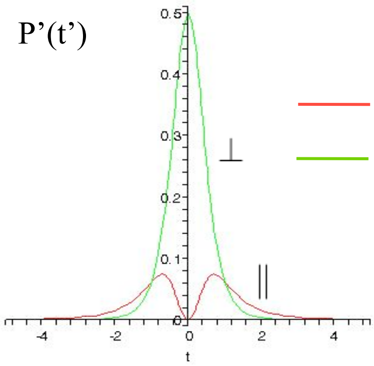
$$= \frac{e^2}{6\pi\epsilon_0 c^3}\left(\frac{Ze^2}{4\pi\epsilon_0 m_e}\right)^2\left[\frac{(\gamma v t')^2}{[(\gamma v t')^2 + b^2]^3} + \frac{(\gamma b)^2}{[(\gamma v t')^2 + b^2]^3}\right]$$

Total radiated power from a single electron-ion encounter

Radiated power: time dependence

$$P' = \frac{e^2}{6\pi\epsilon_0 c^3}\left(\frac{Ze^2}{4\pi\epsilon_0 m_e}\right)^2\left[\underbrace{\frac{(\gamma v t')^2}{[(\gamma v t')^2+b^2]^3}}_{\parallel}+\underbrace{\frac{(\gamma b)^2}{[(\gamma v t')^2+b^2]^3}}_{\perp}\right]$$

P'(t')



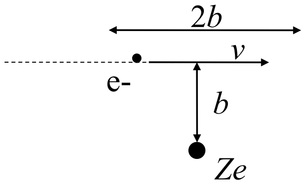
Power from parallel accel.

Power from perpendicular accel.

Dependence on t'

i) Frequency spectrum

Aim: derive frequency spectrum of radiation.
Method: We have acceleration as function of time. Rewrite Larmor formula in terms of acceleration as function of frequency using Parseval's theorem trick. Fourier transform acceleration as function of time and plug in.
Quick estimate:



Time of interaction ~ $2b/v$

Therefore highest frequencies in spectrum

$$\omega' = \frac{2\pi}{T} \simeq 2\pi \times v/2b \sim \frac{\pi v}{b}$$

ie. predicts frequency cutoff order of magnitude

$$\omega' \sim \frac{v}{b}$$

Frequency spectrum by F.T.

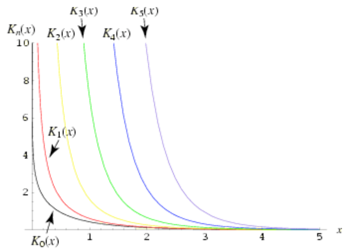
Aims: Frequency spectrum from power as function of time

Method: F.T. accelerations and use in frequency version of Larmor formula.

$$\boxed{\frac{dW}{d\omega} = \frac{e^2}{3\pi\epsilon_0 c^3}|\hat{a}(\omega)|^2}$$

$$\frac{dW'}{dt'} = \frac{e^2}{6\pi\epsilon_0 c^3}\left(\frac{Ze^2}{4\pi\epsilon_0 m_e}\right)^2\left[\frac{(\gamma v t')^2}{[(\gamma v t')^2+b^2]^3} + \frac{(\gamma b)^2}{[(\gamma v t')^2+b^2]^3}\right]$$

$$I'(\omega') = \frac{dW'}{d\omega'} = \frac{Z^2 e^6}{24\pi^4\epsilon_0^3 c^3 m_e^2}\frac{\omega'^2}{\gamma^2 v^4}\underbrace{\left[\frac{1}{\gamma^2}\underset{\parallel}{K_0^2\left(\frac{\omega' b}{\gamma v}\right)} + \underset{\perp}{K_1^2\left(\frac{\omega'^2 b}{\gamma v}\right)}\right]}_{\text{Modified Bessel functions of the second kind}}$$

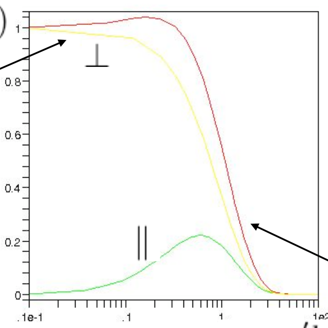


Weisstein, Eric W. "Modified Bessel Function of the Second Kind." From MathWorld--A Wolfram Web Resource.
http://mathworld.wolfram.com/ModifiedBesselFunctionoftheSecondKind.html

Frequency spectrum illustrated

$$I(\omega') = \frac{dW}{d\omega'} = \frac{Z^2 e^6}{24\pi^4\epsilon_0^3 c^3 m_e^2}\frac{\omega^2}{\gamma^2 v^4}\left[\frac{1}{\gamma^2}\underset{\parallel}{K_0^2\left(\frac{\omega' b}{\gamma v}\right)} + \underset{\perp}{K_1^2\left(\frac{\omega' b}{\gamma v}\right)}\right]$$

$I(\omega')$



$y = \omega' b/\gamma v$

Parallel accel.

Perpendicular accel.

Note: Log-linear scale.

Roughly flat spectrum: low frequency limit

$$K_1\left(\frac{\omega' b}{\gamma v}\right) \simeq \frac{\gamma v}{\omega' b}$$

$$\frac{dW}{d\omega'} \simeq \frac{Z^2 e^6}{24\pi^4\epsilon_0^3 c^3 m_e^2}\frac{1}{b^2 v^2}$$

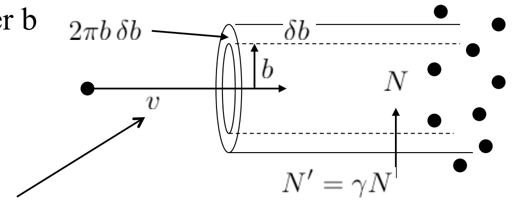
Exponential falloff above $y = \dfrac{\omega' b}{\gamma v} \sim 1$

ii) Multiple encounters

Aim: Derive the frequency spectrum for radiation from electrons with a distribution of velocities interacting with multiple ions.

Method: Integrate single-electron emission over

(1) impact parameter b



(2) Maxwell distribution of electron velocities v

$$N_e(v)\,dv \propto v^2 exp\left(-\frac{m_e v^2}{2kT}\right) dv$$

Gaunt factor

Results: emission coefficient

$$\boxed{4\pi j_\nu = \frac{1}{3\pi^2}\left(\frac{\pi m_e}{6kT}\right)^{1/2}\frac{Z^2 e^6 n n_e}{\epsilon_0^3 c^3 m_e^2}\, g_{\rm ff}(T,\nu)\,\exp(-h\nu/kT)}$$

see Longair 3.5.2, R&L 5.2.

iii) Emission and absorption

Aim: calculate the **absorption coefficient** from the emission coefficient

Method: use **Kirchoff's law** for thermal radiation $\boxed{j_\nu = \alpha_\nu B_\nu(T)}$

to relate emission coeff. j_ν and absorption coeff. α_ν via Planck black-body function $\boxed{\frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}}$

Write emission coefficient as

$$j_\nu = C\frac{Z^2 n n_e}{\sqrt{T}}\, g_{\rm ff}(T,\nu)\,\exp(-h\nu/kT)$$

to find the absorption coefficient

$$\boxed{\alpha_\nu = C\frac{Z^2 n n_e}{\sqrt{T}}\, g_{\rm ff}(T,\nu)\,\frac{c^2}{2h\nu^3}[1-\exp(-h\nu/kT)]}$$

where C depends only on physical constants (c, m_e etc.)

Absorption as function of frequency

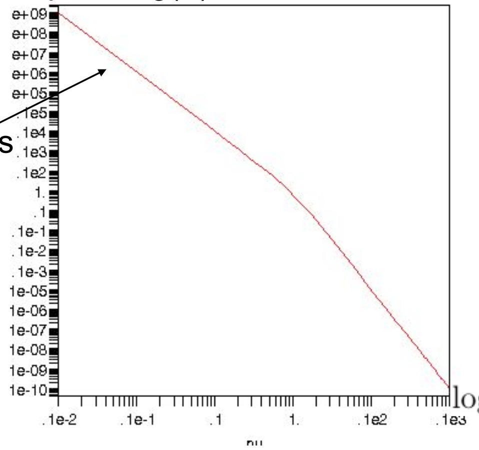
$$\alpha_\nu = C \frac{Z^2 n n_e}{\sqrt{T}} \, g_{\rm ff}(T,\nu) \, \frac{c^2}{2h\nu^3} [1 - \exp(-h\nu/kT)]$$

Absorption log(α)

$h\nu \ll kT$

High absorption at low frequencies

$\alpha_\nu \propto 1/\nu^2$



At high frequencies

$\alpha_\nu \propto 1/\nu^3$

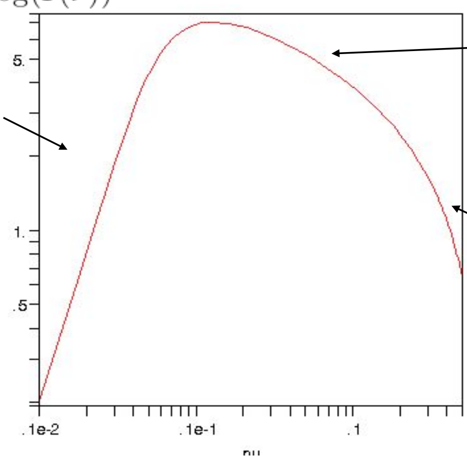
Radio Bremsstrahlung spectrum including self-absorption at low frequency.

Use eqn. of radiative transfer $I(\nu) = B_\nu(T)(1 - e^{-\alpha_\nu S})$
with absorption coeff. as given on previous page

$\log(I(\nu))$

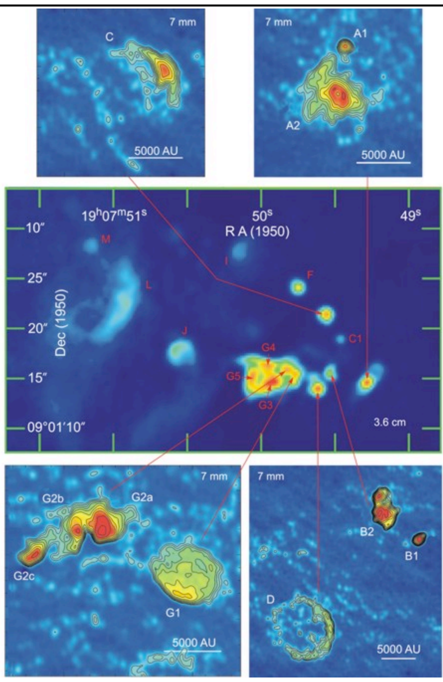
$\tau = \alpha_\nu S \geq 1$

Optically thick
$I \propto \nu^2$

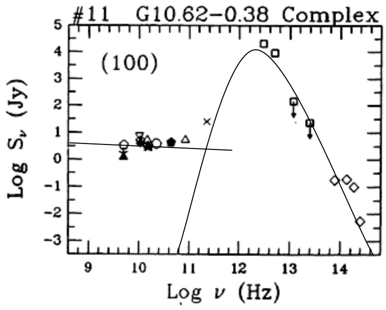


Optically thin
$I \propto \nu^{-0.1}$

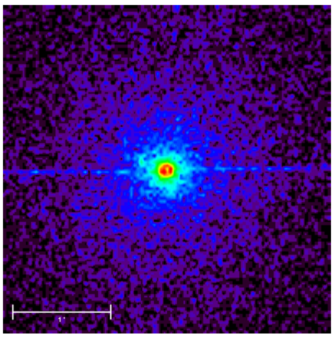
Exponential cutoff for
$h\nu/kT \gg 1$

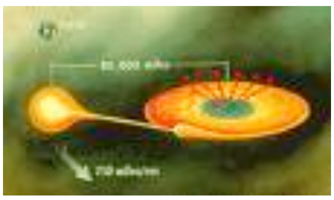


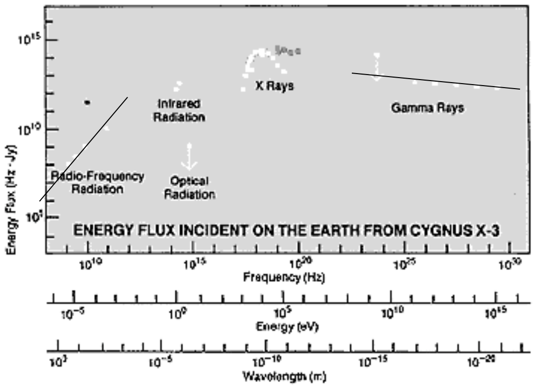
Thermal bremsstrahlung (T ~ 10^4 K):
HII regions around OB stars



X-ray binaries: Cygnus X-3







Synchrotron spectrum

i. Acceleration in magnetic field gives total power via Larmor's formula (lecture 4)

ii. Frequency spectrum via Fourier Transform of P(t) BUT

- The radiation is not isotropic. The magnetic field introduces a preferred direction, exacerbated by relativistic beaming. Have to make the calculation in the lab. frame using the fully relativistic formula for P(t) (remember that Larmor's acceleration^2 version is an approximation for small |v|)

iii. Multiple electrons with a power-law electron energy distribution give a power-law emission coefficient

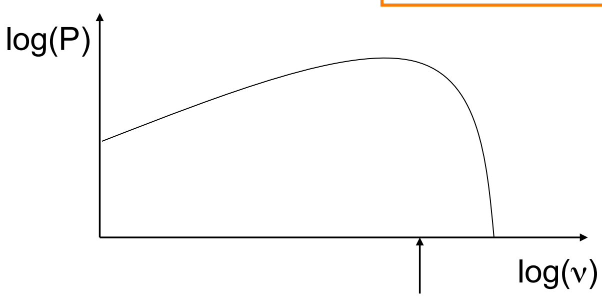
iv. Frequency spectrum from emission and absorption coefficients

Further details: see Longair II ch. 18 or R&L ch. 6

ii. Synchrotron spectrum for single charge

Maximum in frequency spectrum from synchrotron beaming

$$\Delta t \simeq \frac{1}{\gamma^2 \omega_g} \quad \Rightarrow \quad \boxed{\nu_{\max} \sim \gamma^2 \nu_g}$$



$$\nu_c = \frac{3}{2}\left(\frac{c}{v}\right)\gamma^2 \nu_g \sin\theta$$

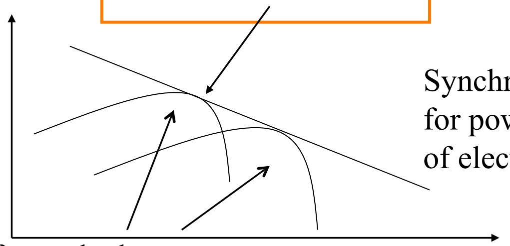
iii. Electron velocity distribution

If electron energy distribution follows a power law

$$N(E)\,dE = KE^{-x}\,dE$$

then the resulting spectrum is also a power law

$$\boxed{j_\nu \propto \nu^{-\frac{1}{2}(x-1)}}$$



iv. Synchrotron self-absorption

If electron energy distribution follows a power law
(note, NOT a thermal distribution)

$$N(E)\,dE = KE^{-x}\,dE$$

Then the emission coefficient is $j_\nu \propto \nu^{-\frac{1}{2}(x-1)}$

and the absorption coefficient $\alpha_\nu \propto \nu^{-\frac{1}{2}(x+4)}$

Applying the equation of radiative transfer: $I_\nu = \dfrac{j_\nu}{\alpha_\nu}(1 - e^{-\alpha_\nu S})$

optically thick $\alpha_\nu S > 1$ $\quad I_\nu = \dfrac{j_\nu}{\alpha_\nu} \propto \nu^{5/2}$

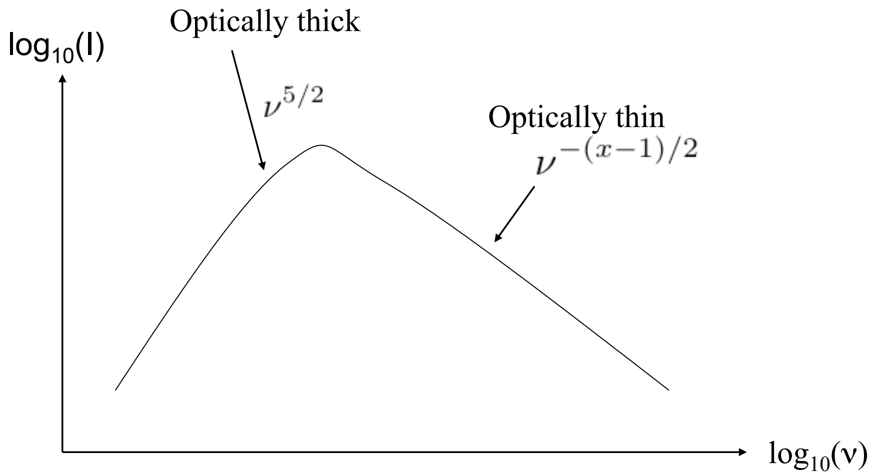
optically thin $\alpha_\nu S \ll 1$ $\quad I_\nu = Sj_\nu \propto \nu^{-\frac{1}{2}(x-1)}$

See Longair II ch. 18 or R&L ch. 6 for derivation.

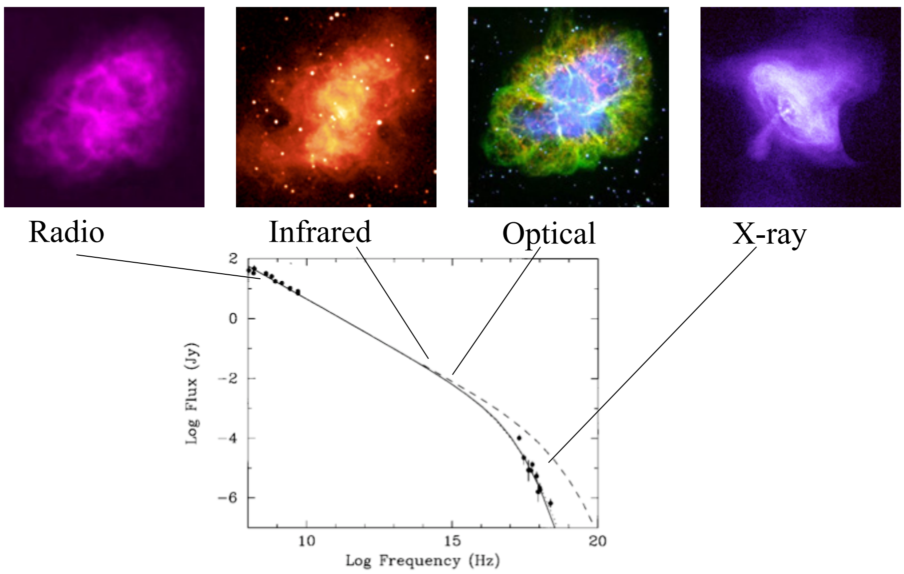
Synchrotron spectrum

from a power-law distribution of electron energies

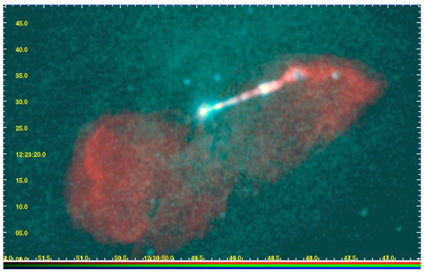
$N(E)\,dE = KE^{-x}\,dE$



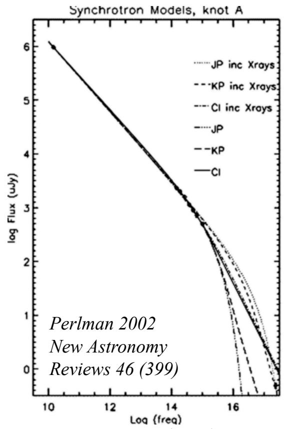
Supernova remnants



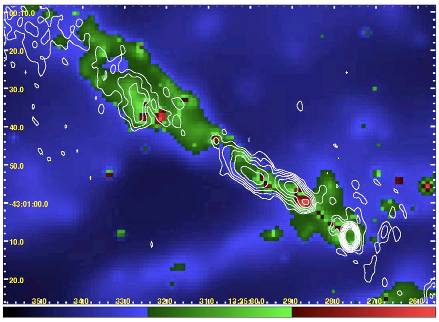
Radio jets



M87 jet in radio (red) / X-ray (green)



M87 spectrum



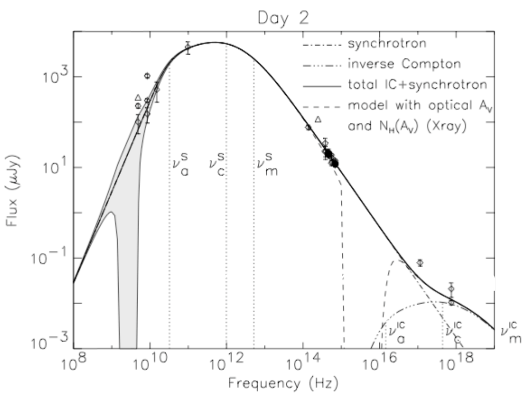
Centaurus A jet in X-ray (colour) and radio (contours)

Images: Schwartz & Harris, http://cxc.harvard.edu/newsletters/news_13/jets.html (2007)

Inverse Compton scattering

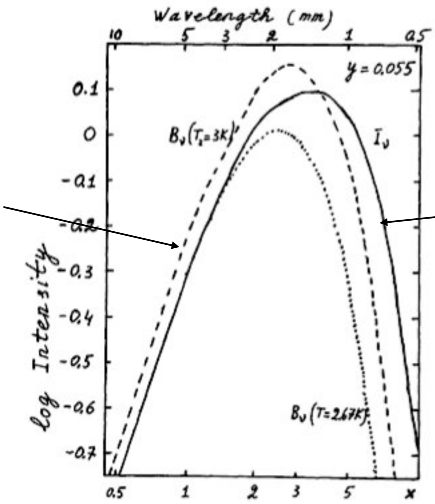
$$v \sim c$$
$$\gamma \hbar \omega \ll m_e c^2$$

Example: Synchrotron self-Compton (SSC)



Inverse Compton and the Sunyaev-Zel'dovich effect

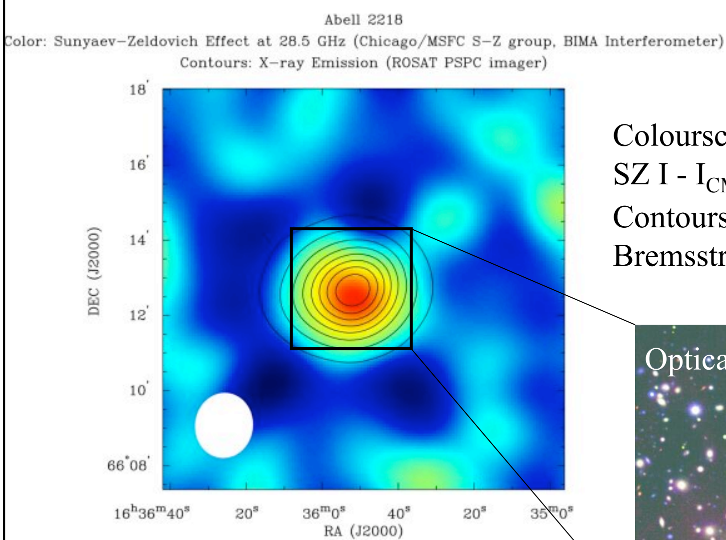
Microwave background (black-body)



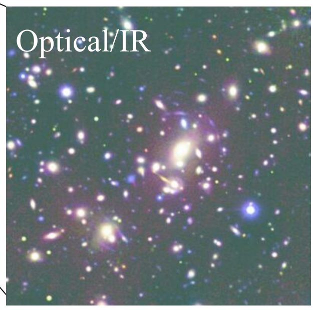
After inverse Compton scattering in hot X-ray cluster gas, photon energies increase

Image: Sunyaev & Zeldovich 1980 ARAA 18 537

S-Z effect: X-ray galaxy cluster



Colourscale:
SZ I - I_{CMB}
Contours: X-ray Bremsstrahlung



Bonamente, Joy et al. 2006