# PHY3145 Topics in Theoretical Physics

# Astrophysical Radiation Processes

2. Accelerated charges emit radiation.

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## Course structure

1. **Radiation basics**. Radiative transfer.
2. **Accelerated charges produce radiation**. Larmor formula. Acceleration in electric and magnetic fields – non-relativistic bremsstrahlung and gyrotron radiation.
3. **Relativistic modifications I**. Doppler shift and photon momentum. Thomson, Compton and inverse Compton scattering.
4. **Relativisitic modifications II**. Emission and arrival times. Superluminal motion and relativistic beaming. Gyrotron, cyclotron and synchrotron beaming. Acceleration in particle rest frame.
5. **Bremsstrahlung and synchrotron spectra.**

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# 2. Accelerated charges emit radiation

(a) Larmor formula for total power

*Examples:*

Coulomb interaction/Bremsstrahlung (non-rel.) (in lectures)

B field / Gyrotron (non-relativistic) (homework)

(b) Angular power pattern

*Examples:*

Thomson scattering (lectures)

Gyrofrequency (homework)

(c) Frequency spectrum via Parseval's theorem

*Example:*

Bremsstrahlung frequency spectrum (lectures, sect. 5)

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# (a) Larmor's Formula for emission from an accelerated charge

The total power radiated from an accelerated, charged particle:

Emitted power in rest frame

charge

acceleration

**Larmor's formula**

$$P = \frac{q^2}{6\pi\epsilon_0 c^3}|a|^2$$

•Valid for non-relativistic velocities (apply in particle rest frame)

**Derivation**: PHY3143 *Advanced Electromagnetism* (Semester II)
Griffiths *Introduction to Electrodynamics* sect.11.2
Longair sect. 3.3.2, Rybicki & Lightman sect. 3.3

# (b) Power pattern of accelerated charge

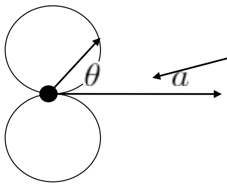
**Aim**: angular dependence of emitted power

Power emitted into an element of solid angle $d\Omega$:

$$\frac{dW}{dt d\Omega} = \frac{q^2}{16\pi^2\epsilon_0 c^3}|a|^2 \sin^2\theta$$

Angle between element of solid angle and acceleration

In rest frame of charge:




Radiation is greatest in direction perpendicular to acceleration.
Examples: Thomson scattering, gyrofrequency



# (c) Larmor formula in terms of frequency

**Aim:** Frequency spectrum from a charge under time-dependent acceleration

**Method:** Energy integrated over time must equal energy integrated over frequency. Use Larmor's formula for power in terms of acceleration at time $t$ and apply Parseval's theorem to get in terms of F.T. of acceleration.

Total energy: equate integral over time with integral over frequency

$$\int_{-\infty}^{\infty} \frac{dW}{dt}\, dt = \int_0^{\infty} \frac{dW}{d\omega}\, d\omega$$

Parseval's theorem:

$$\int_{-\infty}^{\infty} |a(t)|^2\, dt = \int_{-\infty}^{\infty} |\hat{a}(\omega)|^2\, d\omega = 2\int_0^{\infty} |\hat{a}(\omega)|^2\, d\omega$$

See also: R&L 2.3, Longair 3.3.5

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Using Larmor formula and applying Parseval's theorem:

$$\int_{-\infty}^{\infty} \frac{dW}{dt}\,dt = \int_{-\infty}^{\infty} \frac{e^2}{6\pi\epsilon_0 c^3}|a(t)|^2\,dt = \int_0^{\infty} \frac{2e^2}{6\pi\epsilon_0 c^3}|\hat{a}(\omega)|^2\,d\omega = \int_0^{\infty} \frac{dW}{d\omega}\,d\omega$$

Equating:

$$\frac{dW}{d\omega} = \frac{e^2}{3\pi\epsilon_0 c^3}|\hat{a}(\omega)|^2$$

Larmor formula in terms of frequency

NB less by factor of 2 than total power formula

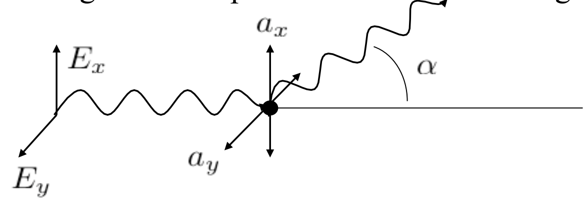


# (b) Power pattern example: Thomson scattering

$$\hbar\omega \ll m_e c^2$$
$$v \ll c$$

**Aim**: Derive angular dependence of power for classical scattering of EM field by a non-relativistic electron.

**Method:** Treat photon as EM wave accelerating electron. Consider acceleration in plane of scattering and perpendicular to plane of scattering separately. Use Larmor's formula (with angular dependence) to get emitted power as a function of angle.




*x*-component of accel. (in plane of $\alpha$) => power $\propto \cos^2\alpha$

*y*-component of accel. (perp. to plane of $\alpha$) => power independent of $\alpha$

**Result: (derivation in lectures)**

$$\frac{dW}{dt d\Omega} = \frac{e^4}{16\pi^2\epsilon_0 c^3 m_e^2}(|\langle E_x\rangle|^2\cos^2\alpha + |\langle E_y\rangle|^2)$$

# Thomson cross section

$$\hbar\omega \ll m_e c^2$$
$$v \ll c$$

**Aims**: Differential and total scattering cross-section for Thomson scattering

**Method:** Differential scattering cross-section is defined as

$$\text{Differential cross-section} = \frac{\text{(power scattered into solid angle)}}{\text{(incident power per unit area)}}$$

Total scattering cross-section given by integral over solid angle.

**Results (derivation in lectures):**

**Differential Thomson cross section**

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}(1+\cos^2\alpha)$$

**Thomson cross-section**

$$\sigma = \frac{8\pi}{3} r_e^2$$

using the **classical electron radius**

$$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2}$$