Migration and Accretion of Protoplanets in 2D and 3D Global Hydrodynamical Simulations

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Abstract. Planet evolution is tightly connected to the dynamics of both distant and close disk material. Hence, an appropriate description of disk-planet interaction requires global and high resolution computations, which we accomplish by applying a Nested-Grid method. Through simulations in two and three dimensions, we investigate how migration and accretion are affected by long and short range interactions. For small mass objects, 3D models provide longer growth and migration time scales than 2D ones do, whereas time lengths are comparable for large mass planets.

1. Introduction

Migration has entered the puzzling scenario of planetary formation as the favorite mechanism advocated to explain the extremely short orbital period of many extrasolar planets.

It is known that any planet-like body is forced to adjust its distance from the central star because of gravitational interactions with the circumstellar material. However, the dispute about how fast migration proceeds is far from being over. Numerical methods have been employed to evaluate gravitational torques exerted on embedded planets. We have performed a series of simulations modeling both two and three dimensional disks, varying the mass M_p of the protoplanet in the range from 1 Earth-mass to 1 Jupiter-mass.

The physics of the problem demands that the flow in the protoplanet's neighborhood should be accurately resolved. In order to achieve sufficient resolution, even for very small planetary masses, we use a *Nested-Grid* technique (D'Angelo, Henning, & Kley 2002). This paper addresses the issues of flow circulation around protoplanets, orbital migration, and mass accretion.

2. Numerical Model

We assume the protostellar disk to be a viscous fluid (viscosity $\nu = 10^{15}\ \mathrm{cm^2\,s^{-1}}$) and describe it through the Navier-Stokes equations (Kley, D'Angelo, & Henning
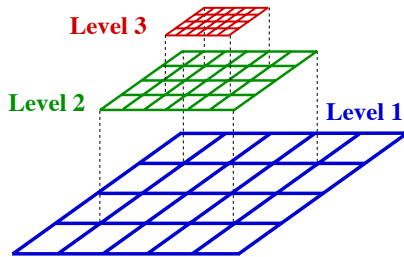
Figure 1.   Sketch of a 2D, three-level Nested-Grid system. Each of the first two grid levels hosts a finer grid, allowing for an increasing accuracy. In these computations up to 7 grid levels were employed.
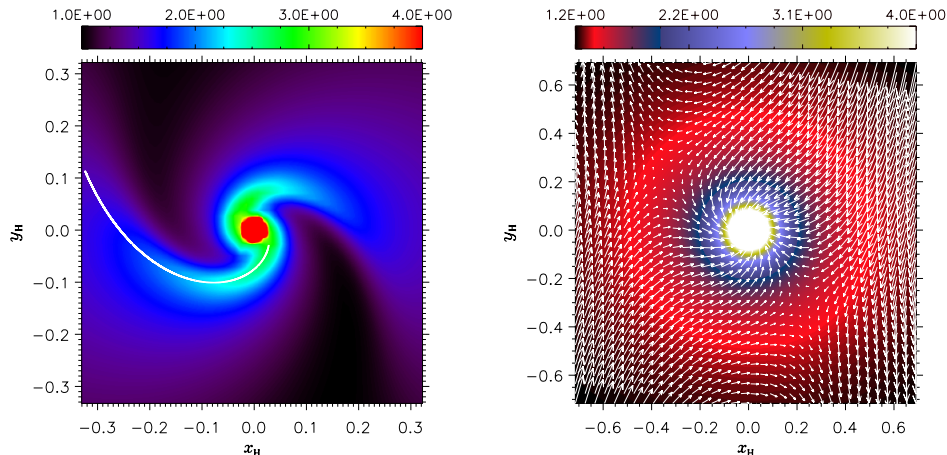

Figure 2.   Surface density around a Uranus (*left*) and an Earth (*right*) mass protoplanet. Axis scales are in Hill coordinates. In physical units, $\Sigma = 4$ corresponds to $256 \; \mathrm{g \, cm^{-2}}$.

2001; D'Angelo et al. 2002). The set of equations is integrated over a grid hierarchy, as shown in Figure 1. The planet is supposed to move on a circular orbit at $r_p = 5.2$ AU around a solar-mass star. The disk has an aspect ratio $h = 0.05$ and the mass within the simulated region is $M_D = 3.5 \times 10^{-3} \; M_\odot$.

3.   2D Simulations of Uranus and the Earth

A circumplanetary disk forms inside the Roche lobe of massive as well as low-mass protoplanets. Such structures are characterized by a two-arm spiral wave perturbation. Indicating with l the distance from the planet normalized to r_p, for a wide range of planetary masses the spiral pattern can be approximated to

$$\Theta - \Theta_0 = 2 \, k \left[l_0^{-1/2} - l^{-1/2} \right], \tag{1}$$

where $k = \zeta \, \sqrt{M_p/M_*}/h$ and $\zeta \approx 1$. The ratio $k/\sqrt{l}$ represents the Mach number of the circumplanetary flow. Figure 2 (left panel) demonstrates how equation (1) fits to the spiral perturbation around a Uranus-mass planet.

Even an Earth-mass planet induces two weak spirals, which wrap around the star for 2π, but no circumplanetary disk is observed (Figure 2, right panel).
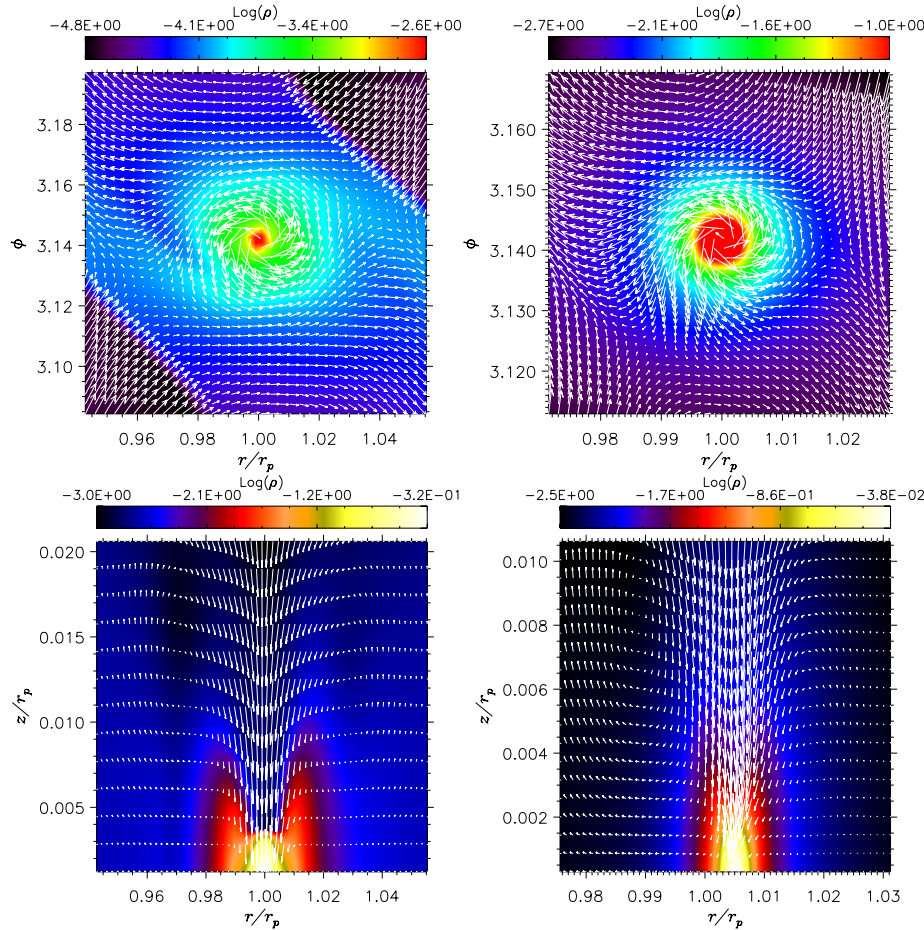
Figure 3.   Density and velocity field in the equatorial plane (*top*) and in a vertical plane containing the planet (*bottom*). *Left panels* refer to Saturn, *right panels* to Neptune. In the plot, $\rho = 0.1$ corresponds to 4.2×10^{-10} g cm^{-3}. Maximum velocities are on the order of 3 km s^{-1}.

4.   3D Simulations of Saturn and Neptune

A more complete description of the flow near protoplanets is provided by 3D computations (D'Angelo, Kley, & Henning 2003). The major differences between 2D and 3D modeling arise in the vicinity of the planet because the latter can account for the vertical circulation in the circumplanetary disk. Instead, the two geometries decently agree on length scales larger than the disk scale height.

Two examples of our simulations are illustrated in Figure 3. The images represent the logarithm of the density close to a Saturn-mass planet (left) and Neptune-mass planet (right), in two orthogonal planes (see Figure caption for details). The velocity field is overplotted to display the flow features. From the top panels it is clear that the spiral perturbations are weaker and more open than in 2D simulations. The bottom panels indicate the presence of vertical shock fronts, which are generally located outside the Hill sphere of the protoplanet.
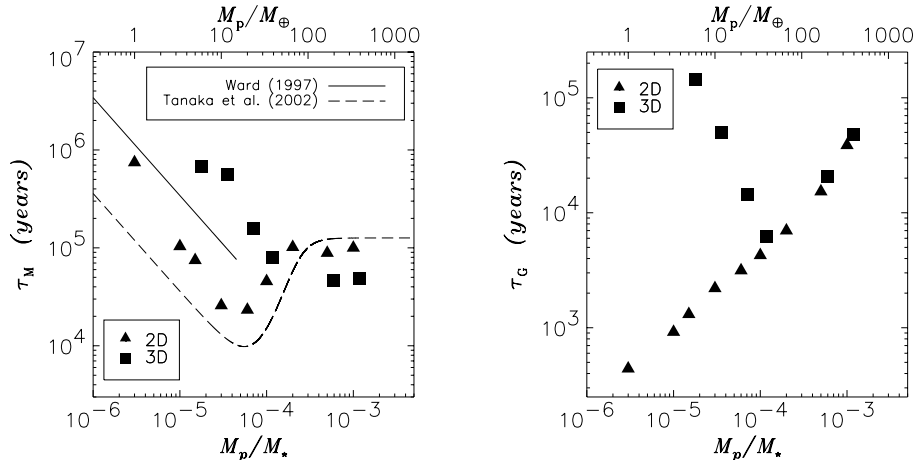
Figure 4.   Migration (*left*) and growth (*right*) time scale for different planetary masses. Migration is compared to two analytical models: Ward's one doesn't include corotation torques whereas the other does.

5.   Migration and Accretion

In general, torques exerted by disk material cause a protoplanet to migrate toward the star (Ward 1997). Yet, nearby matter can be very efficient at slowing down its inward motion (Tanaka, Takeuchi, & Ward 2002; D'Angelo et al. 2002). The migration time scale can be defined as $\tau_M \equiv r_p/|\dot{r}_p|$, where the migration drift $\dot{r}_p$ is directly proportional to the total torque acting on the planet. Since we also measure the rate $\dot{M}_p$ at which the planet accretes matter from its surroundings, an accretion time scale can be introduced as well: $\tau_G \equiv M_p/\dot{M}_p$.

Some of our 2D and 3D outcomes for both time scales are shown in Figure 4.

6.   Summary

Circumplanetary disk forms around protoplanets. The spiral wave pattern which marks such disks is less accentuated when the full 3D structure is simulated. Vertical shock fronts develop outside the Hill sphere of the planet.

The estimated values of τ_M in 3D are longer than those predicted by analytical linear theories because of non-linearity effects. When $M_p \lesssim 30\ M_\oplus$, both τ_M and τ_G are longer in 3D computations then they are in 2D ones.

References

D'Angelo, G., Henning, Th., & Kley, W. 2002, A&A, 385, 647

D'Angelo, G., Kley, W., & Henning, Th. 2003, ApJ, in press

Kley, W., D'Angelo, G., & Henning, Th. 2001, A&A, 547, 457

Tanaka, H., Takeuchi, T., & Ward, W. 2002, ApJ, 565, 1257

Ward, W. 1997, Icarus, 126, 261