

# Dynamics of Terrestrial Atmospheres

From Earth to Titan, in parameter space  
with some discussion of superrotation

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January 15, 2012

# Outline

- 1 The Earth's Atmosphere (review, about half the talk)
- 2 Variations of parameters, rotation rate, planetary radius etc.
  - ▶ With particular application to superrotation and Titan

# The Other Book

## Atmospheric and Oceanic Fluid Dynamics

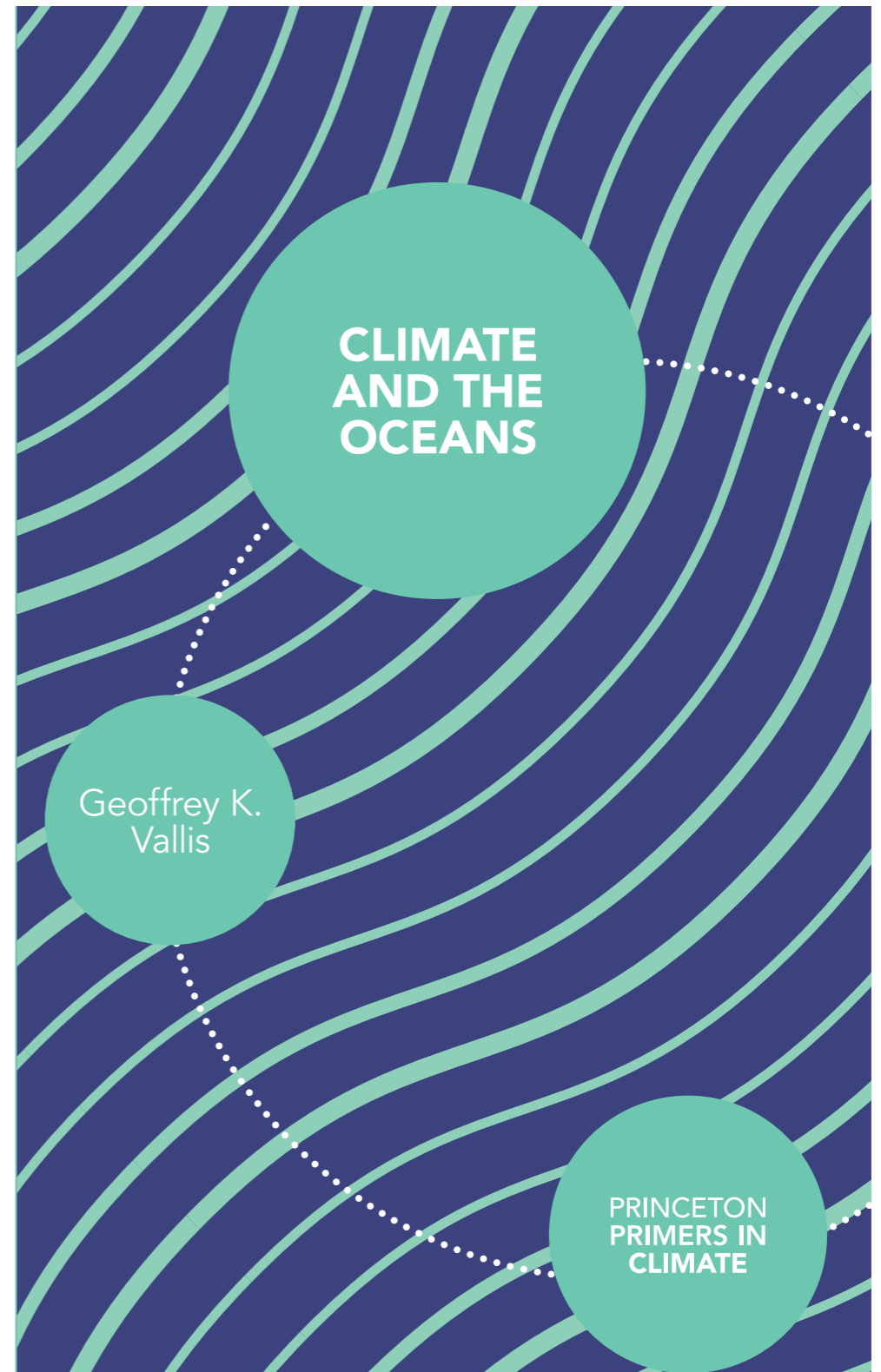
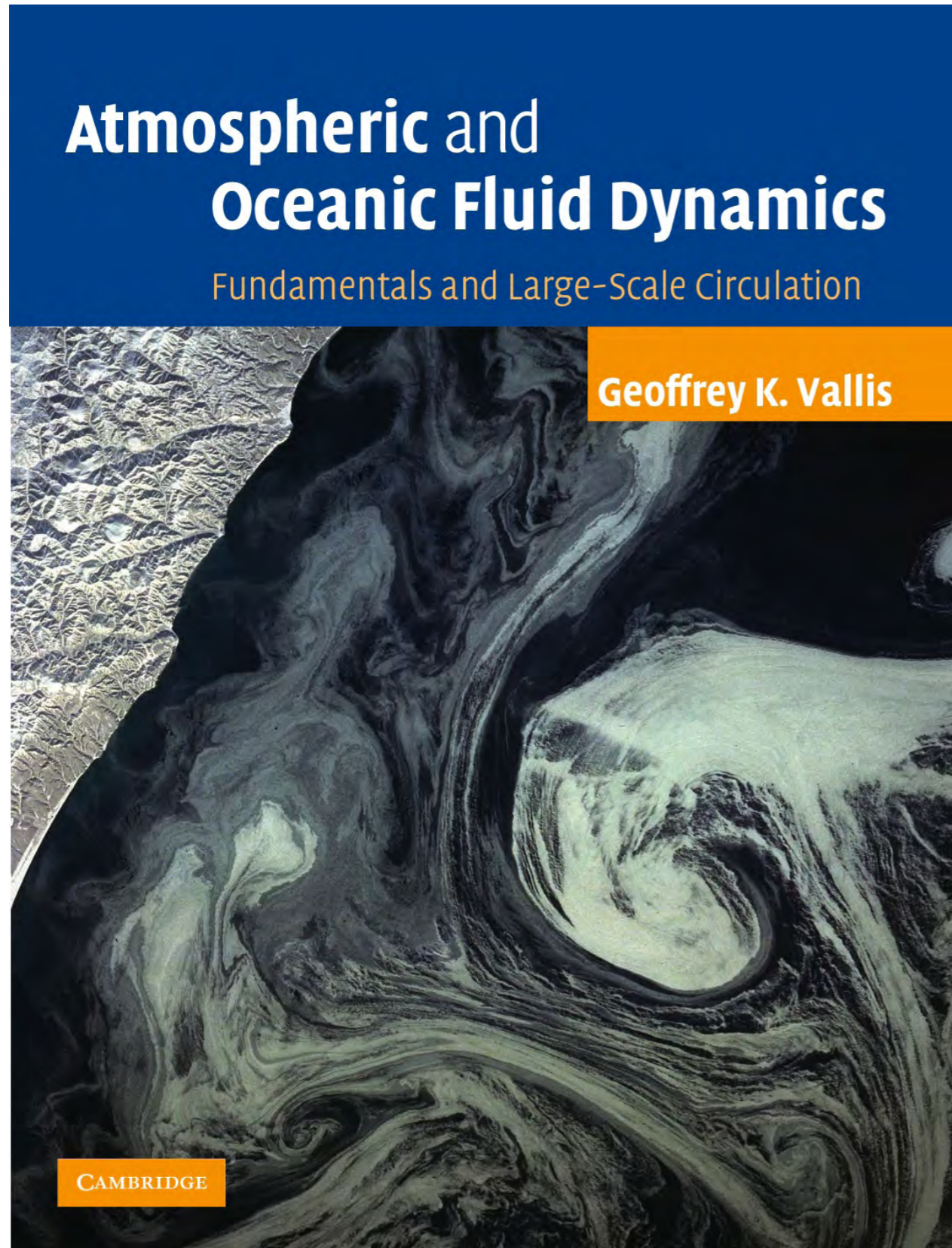
Fundamentals and Large-Scale Circulation

Geoffrey K. Vallis

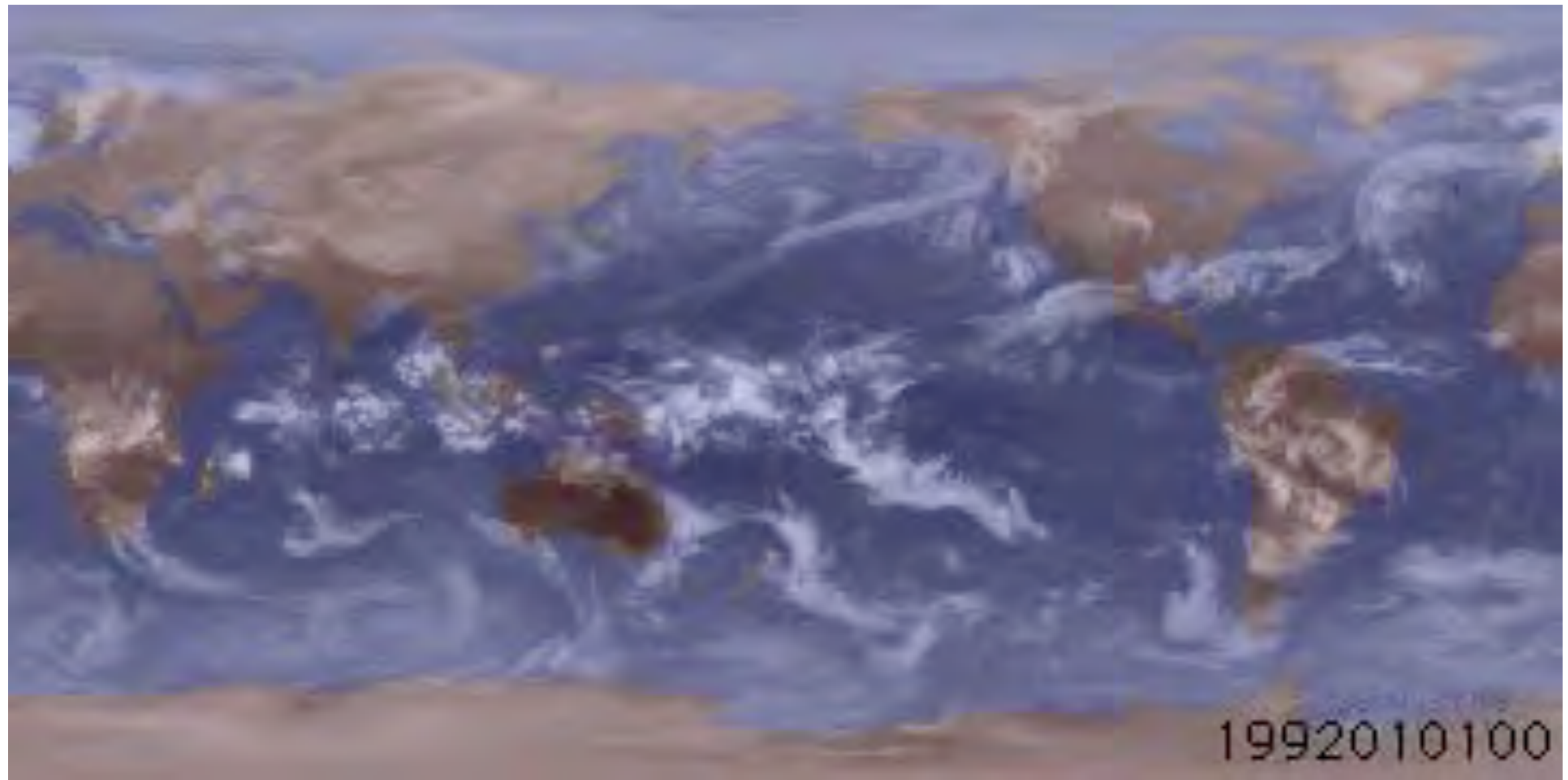
CAMBRIDGE

A satellite image of Earth's atmosphere and oceans, showing complex fluid dynamics patterns. The image features swirling clouds and ocean currents, with a prominent cyclone or storm system visible in the lower right quadrant. The colors range from dark blues and greys to lighter greys and whites, highlighting the intricate structures of the atmosphere and ocean.

# The Other Book

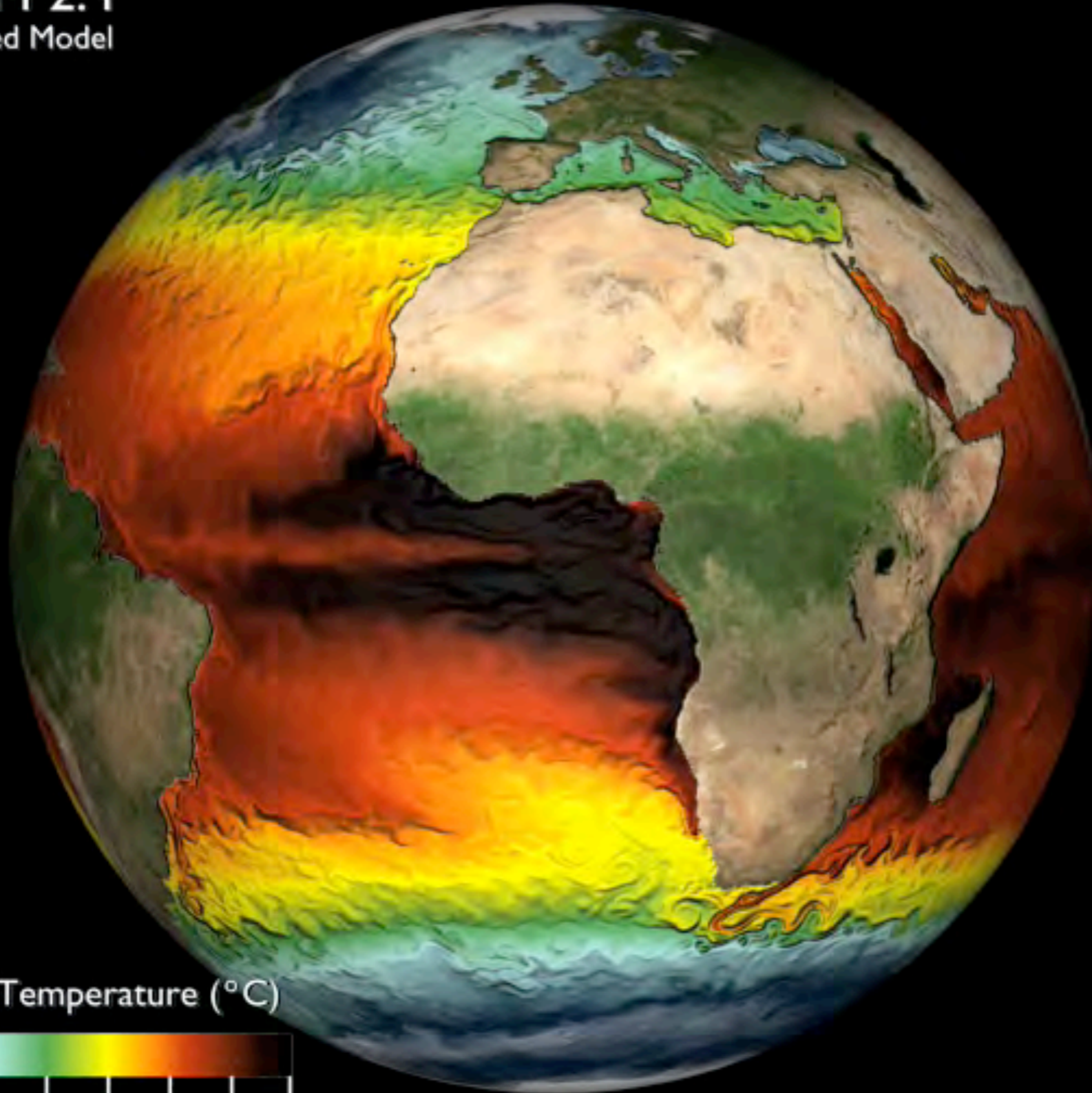




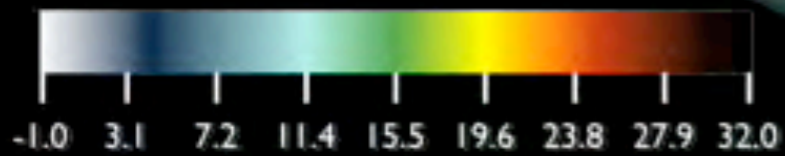


# GFDL CM 2.4

Hi-Res Coupled Model



Sea Surface Temperature ( $^{\circ}\text{C}$ )



# *The Earth*

To begin at the beginning:

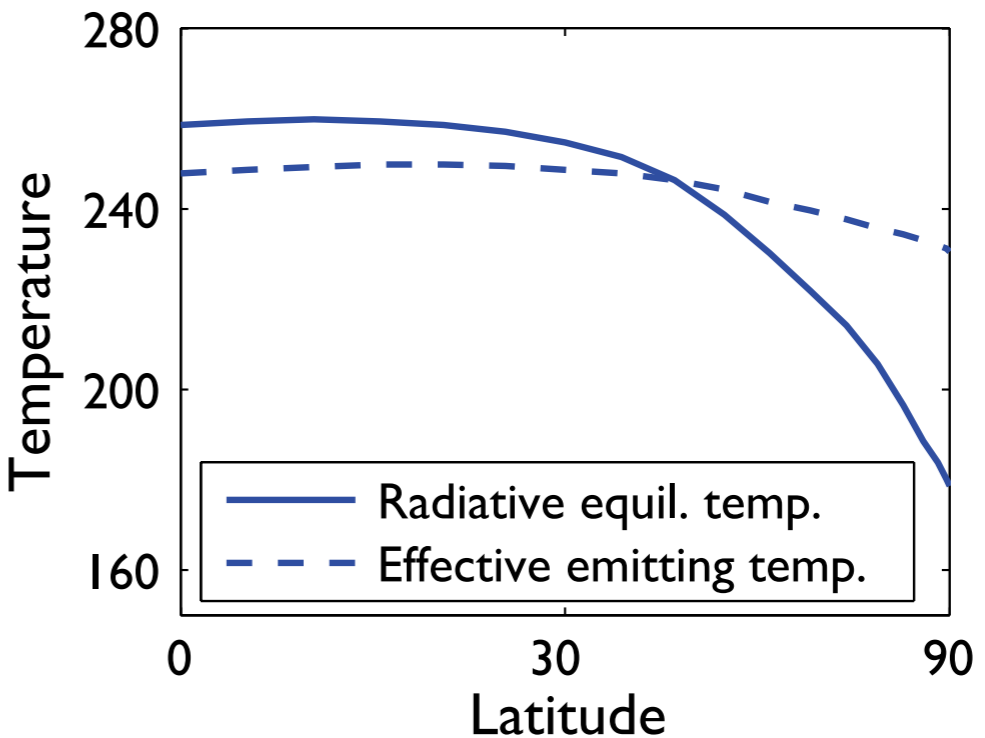
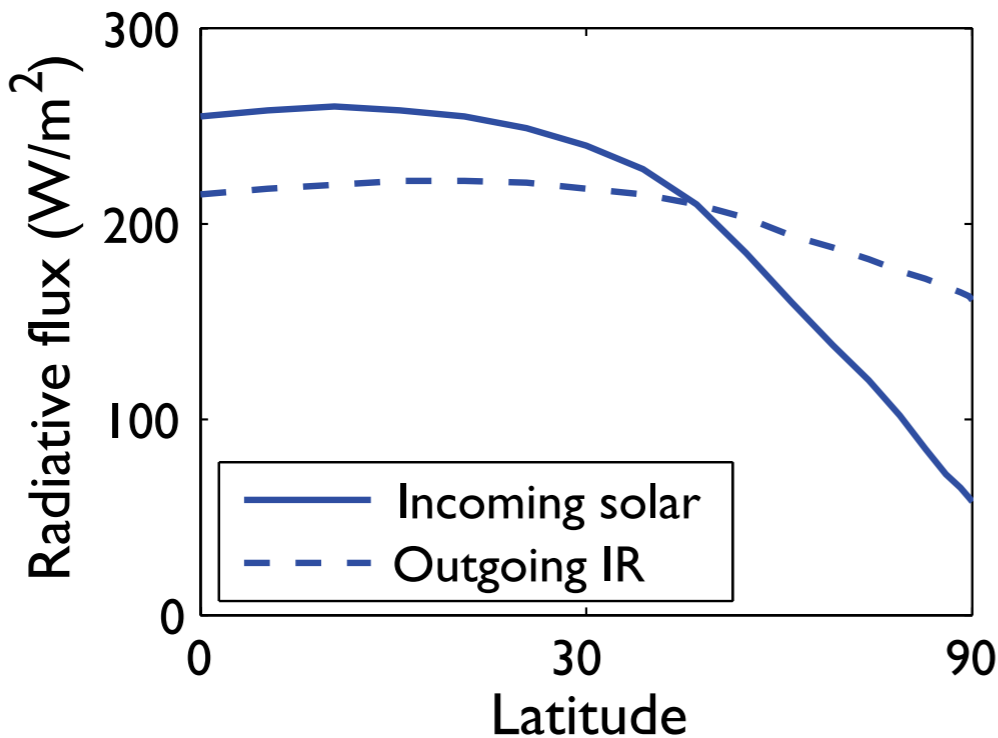
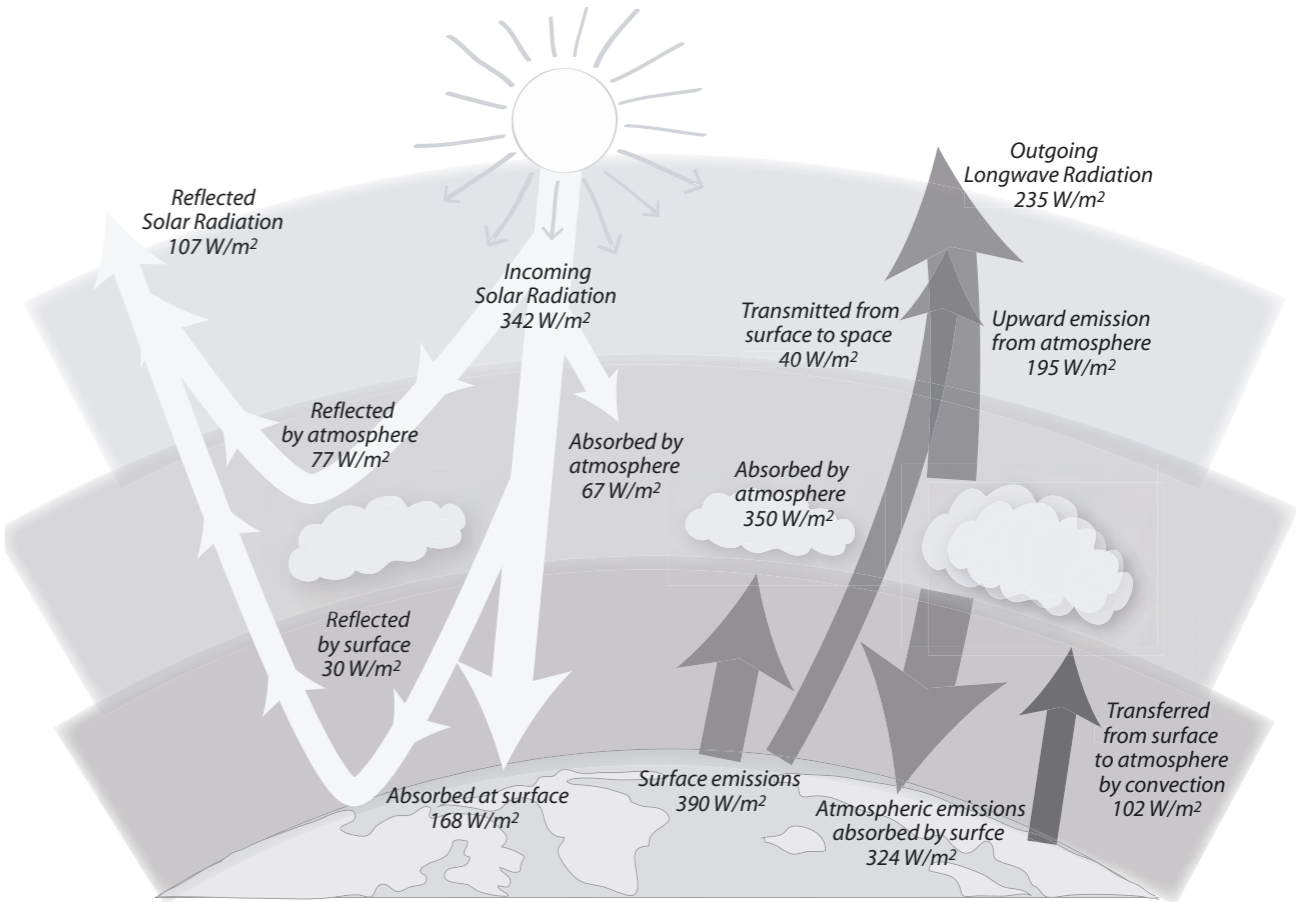
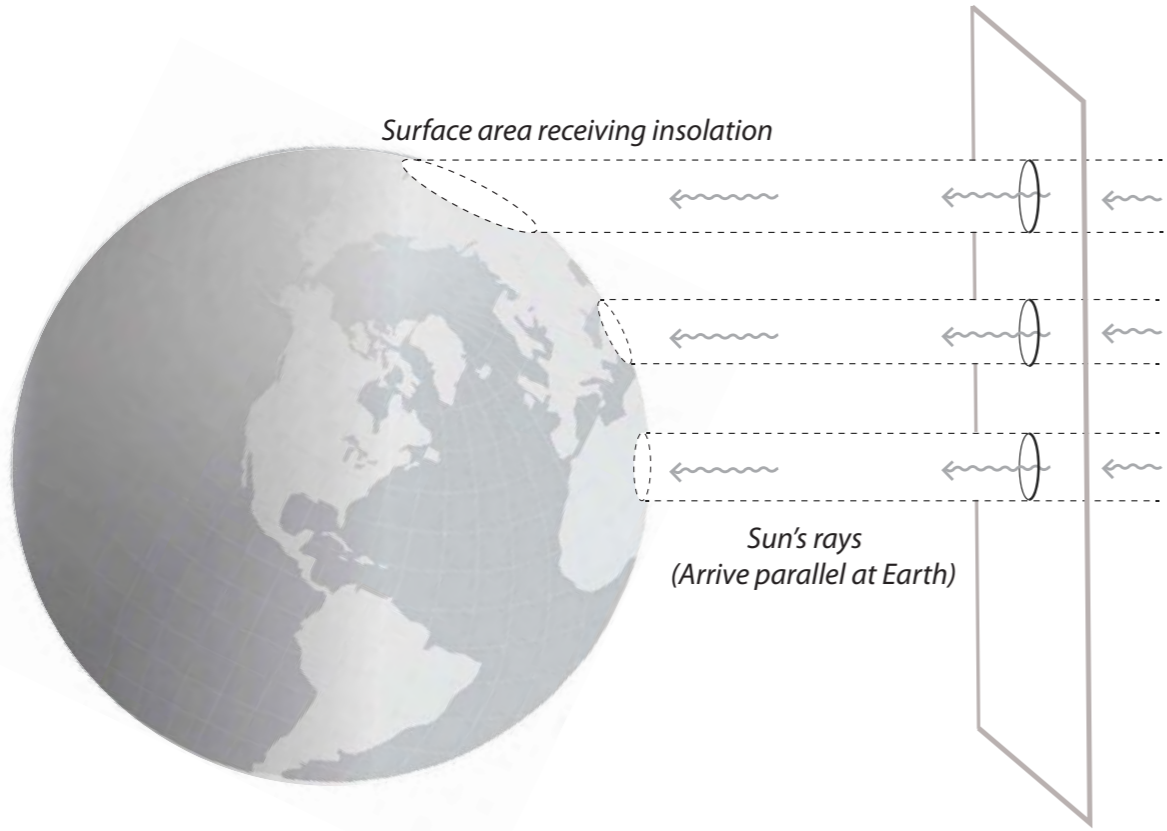
The Earth's atmosphere is affected by:

- 1 Rotation, and secondarily differential rotation (beta effect).
- 2 Differential heating (both in latitude and altitude).

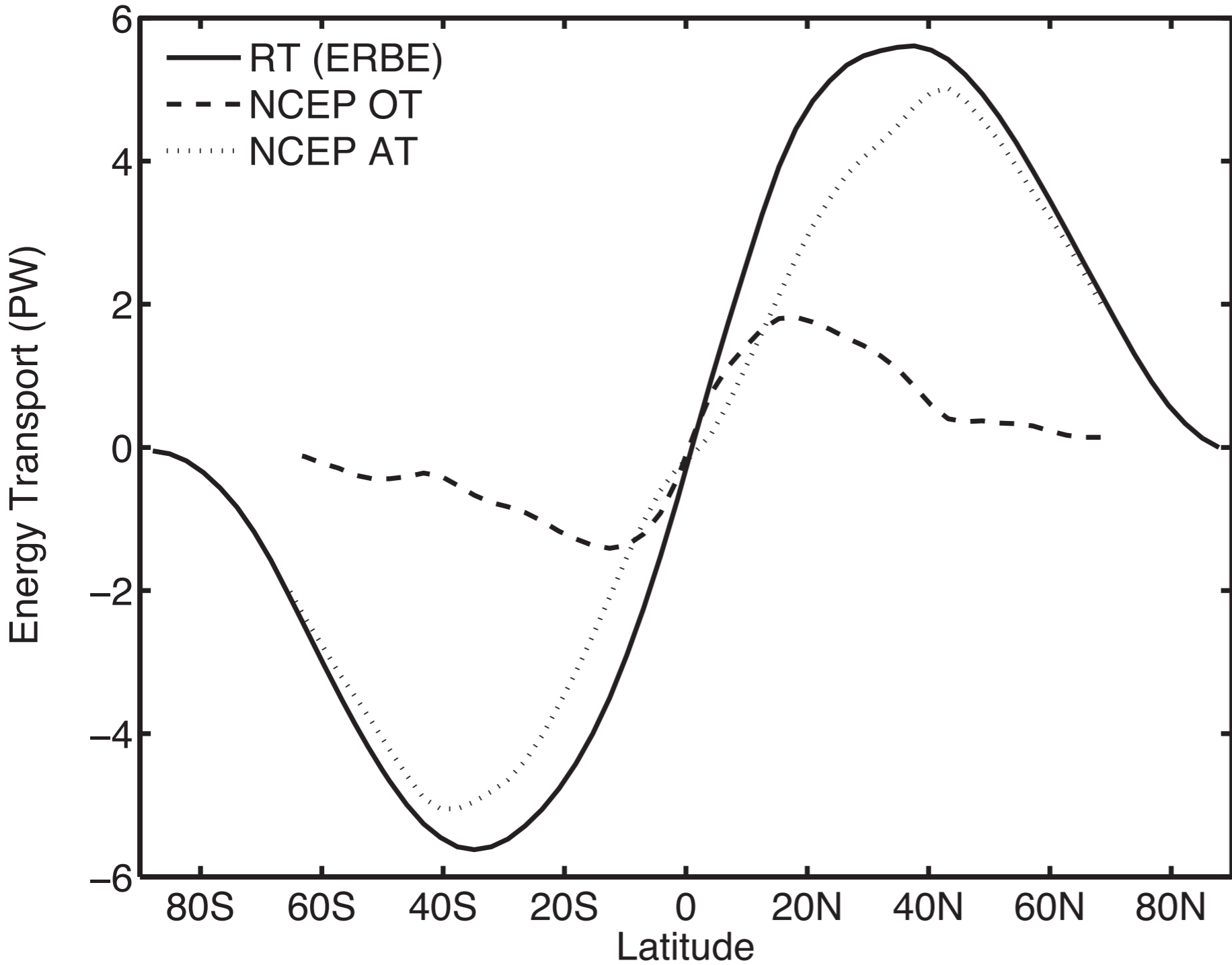
In so far as there are no phase changes (rain, ice, snow, etc) the absolute temperature is not so important and the Earth could be twice as far from the Sun with a similar circulation.



# Radiation Budget



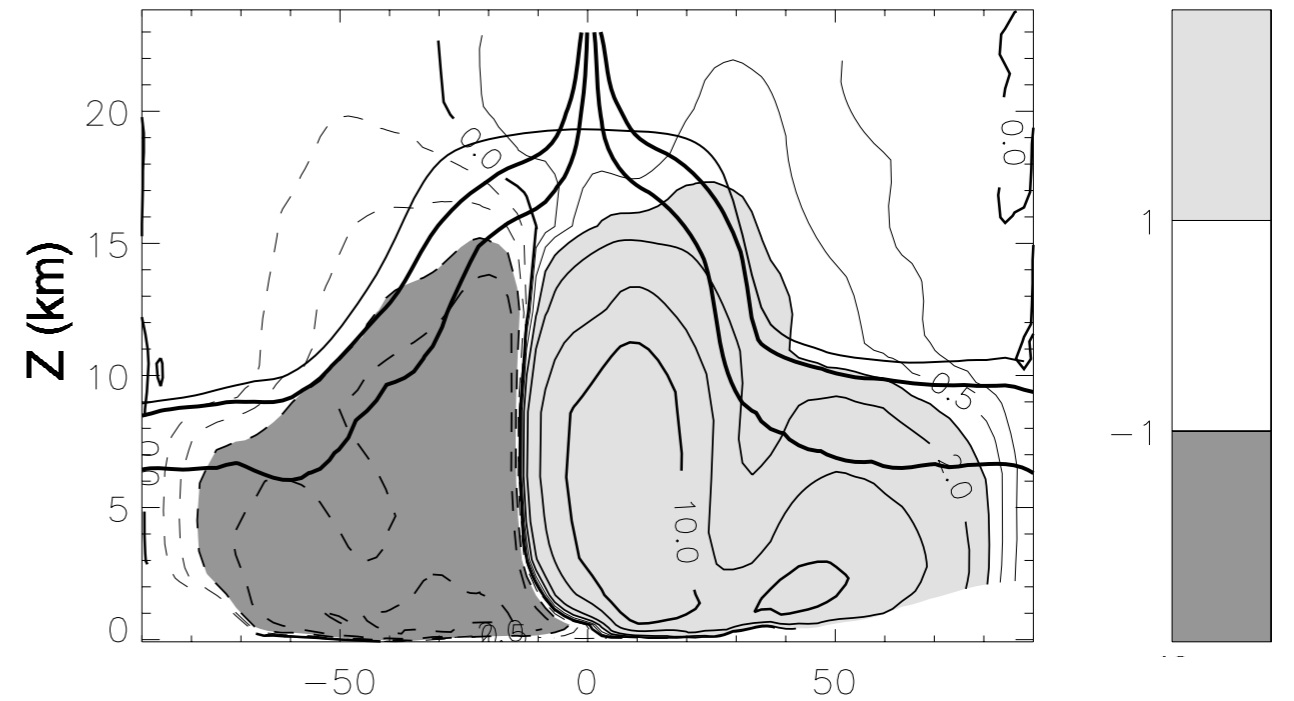
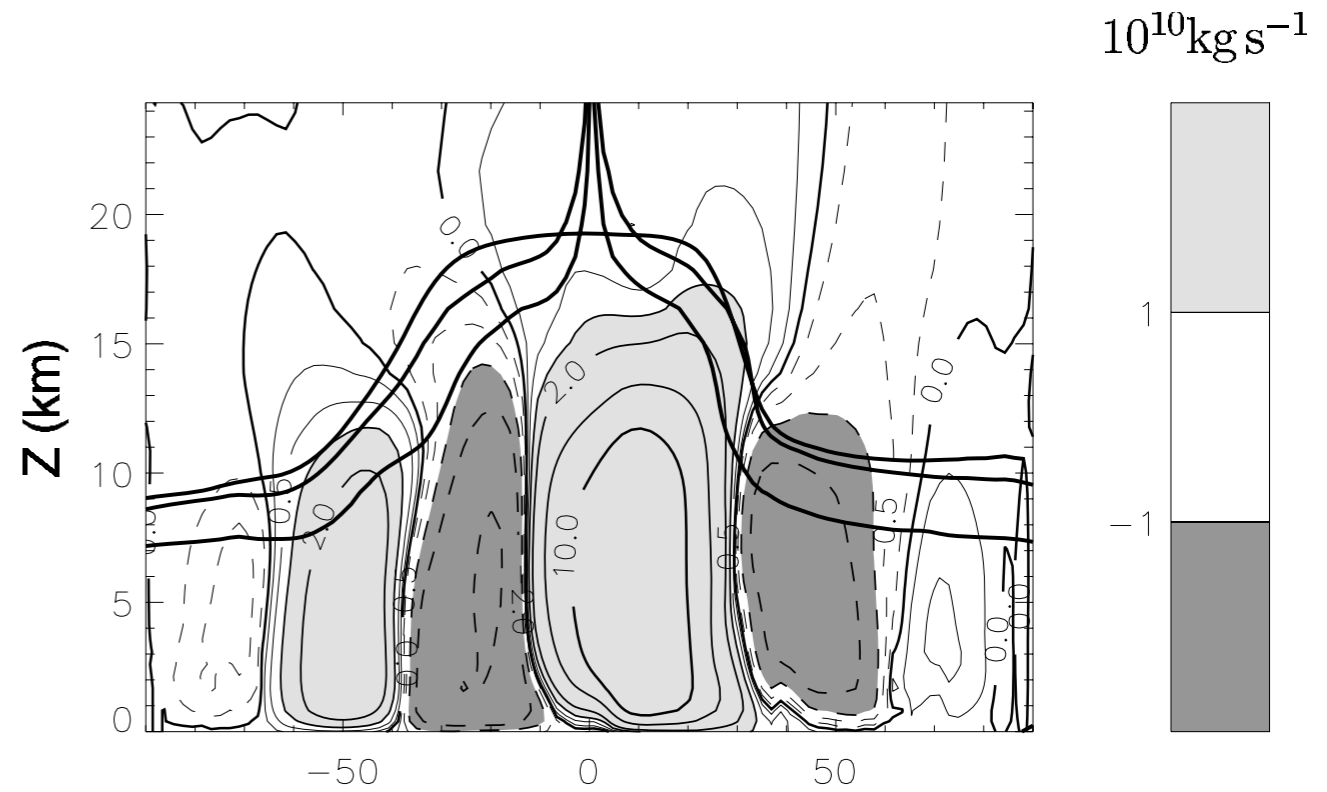
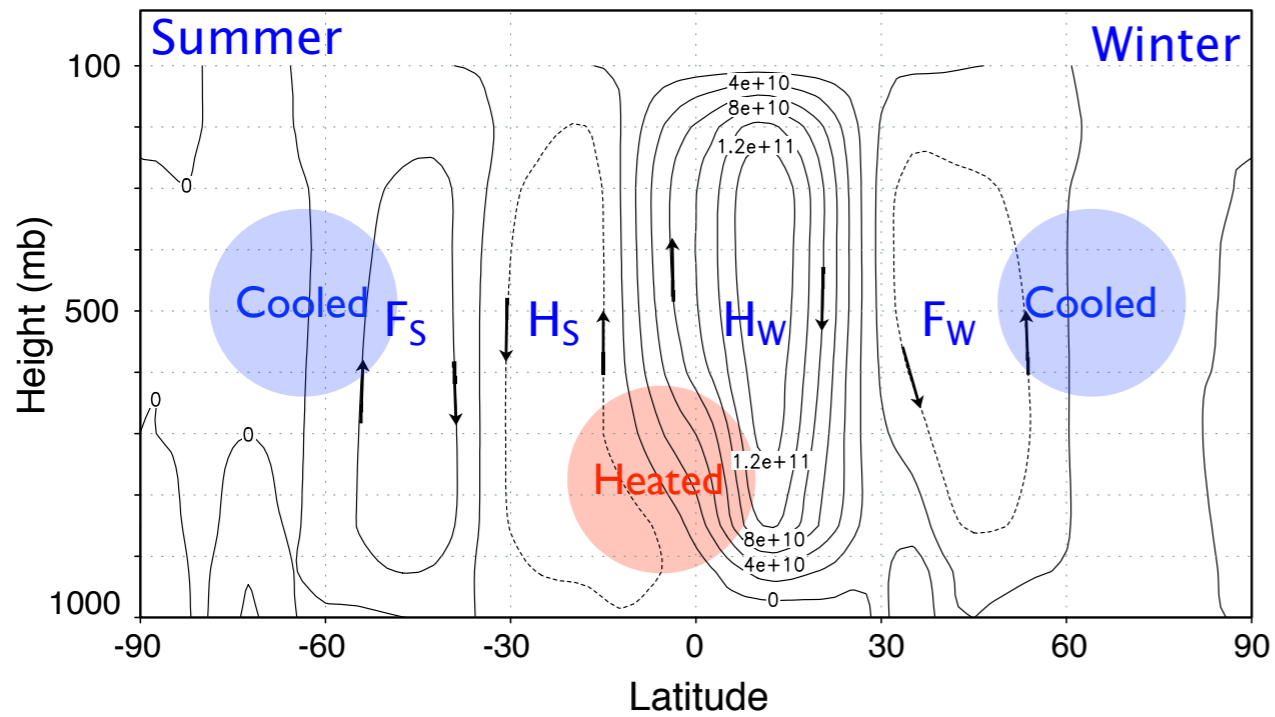
# Heat Transport



From  
Trenberth and Caron

# Circulation and Dynamics

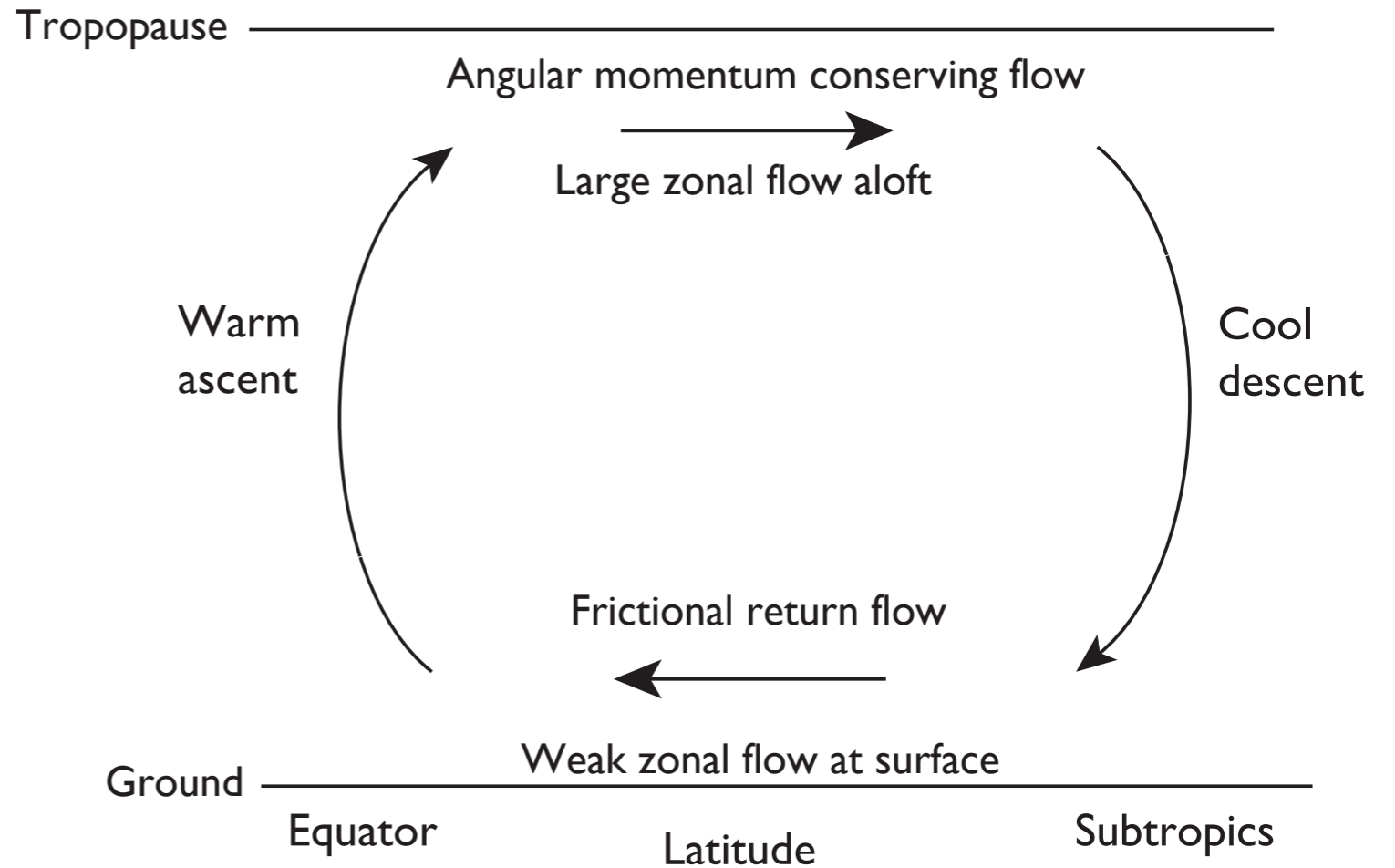
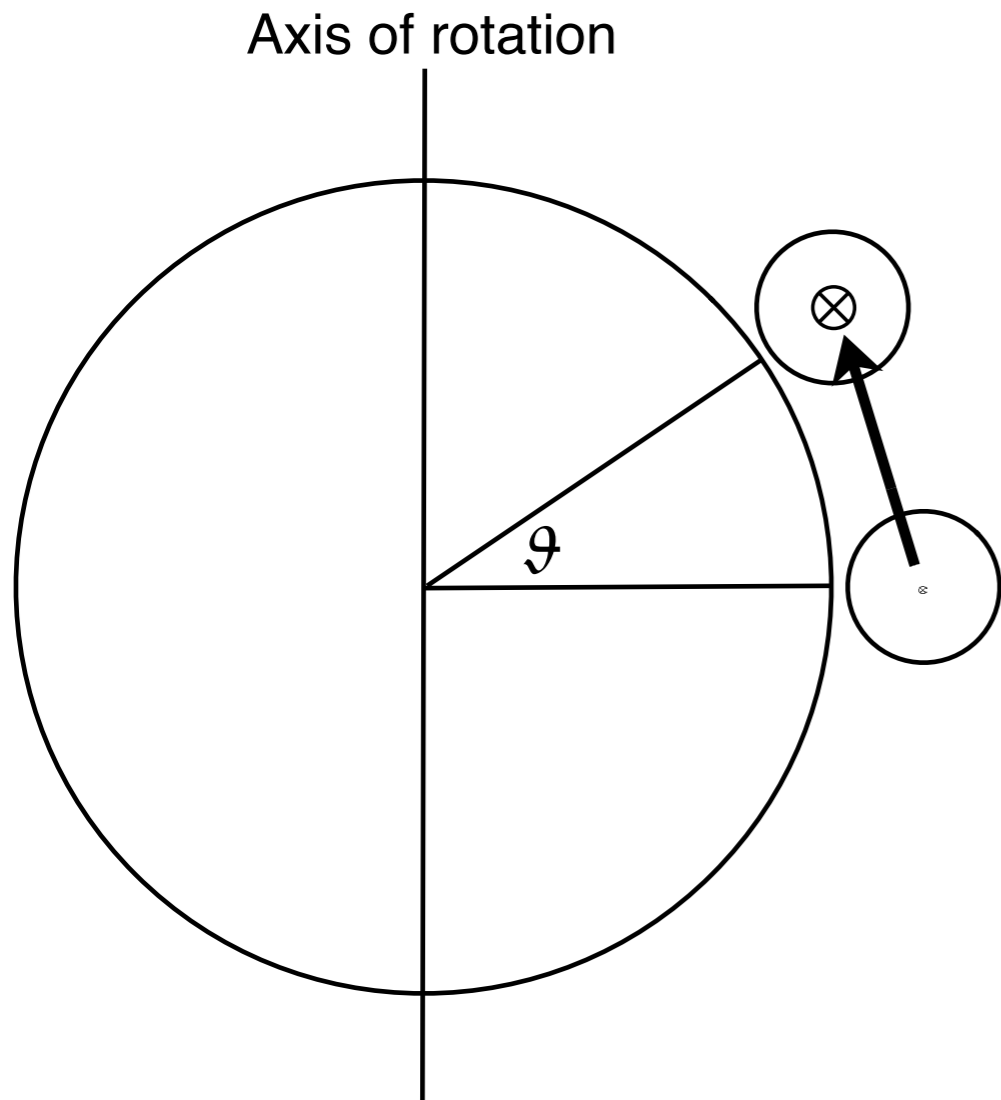
## Meridional Overturning Circulation



A heat engine in part - heated at equator, cooled at pole, with the circulation produced by that differential heating. Atmosphere transparent to solar radiation, opaque to IR - so heating occurs at the ground, at a lower level than the cooling.

# Hadley Cell Dynamics

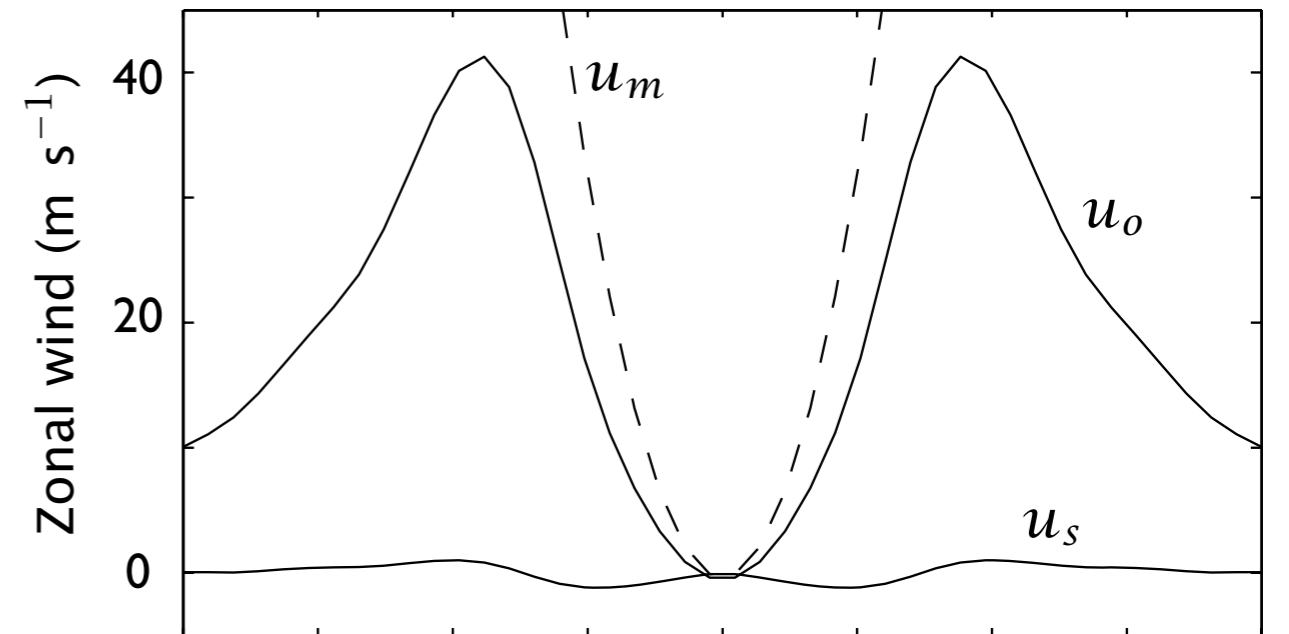
Part angular momentum conservation, part eddy eddy dynamics.



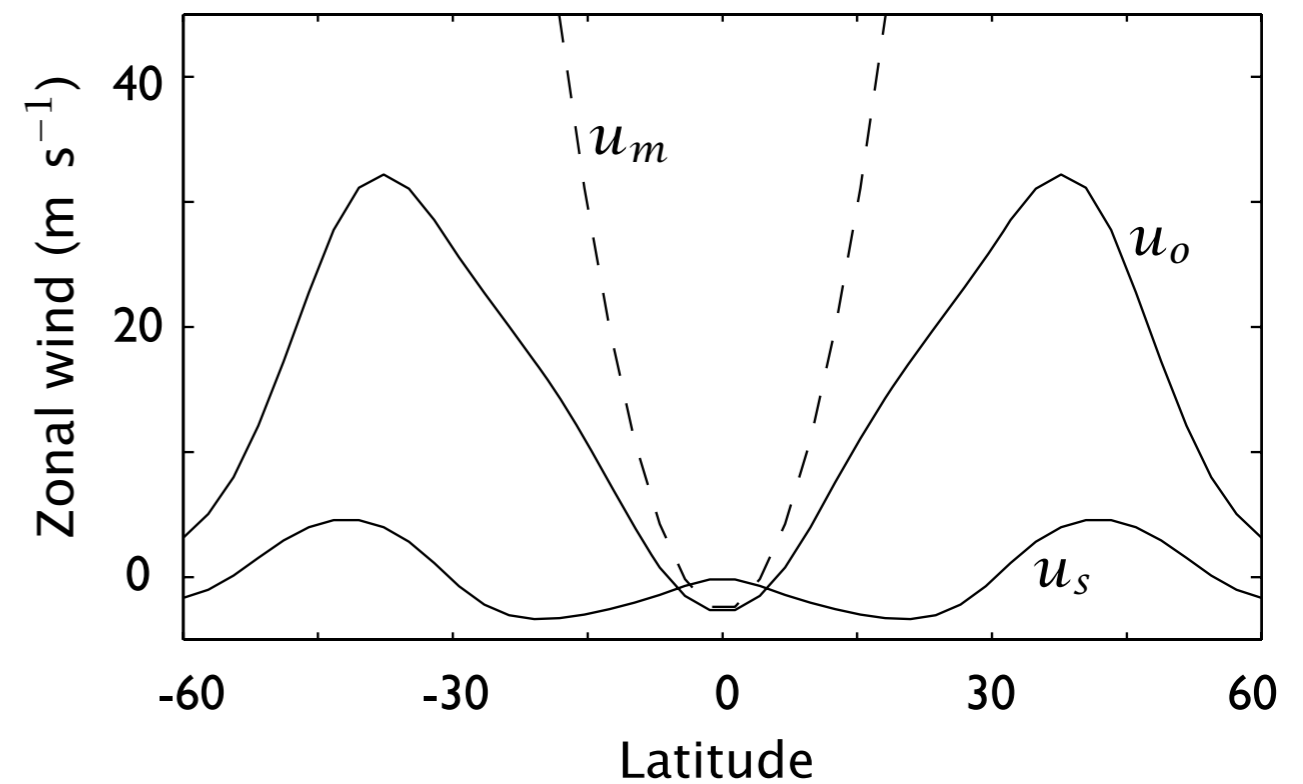
Air moving polewards moves closer to axis of rotation, and zonal flow must increase to maintain angular momentum conservation

# Numerical Experiments on Hadley Cell

Axi-symmetric model, nearly angular momentum conserving.



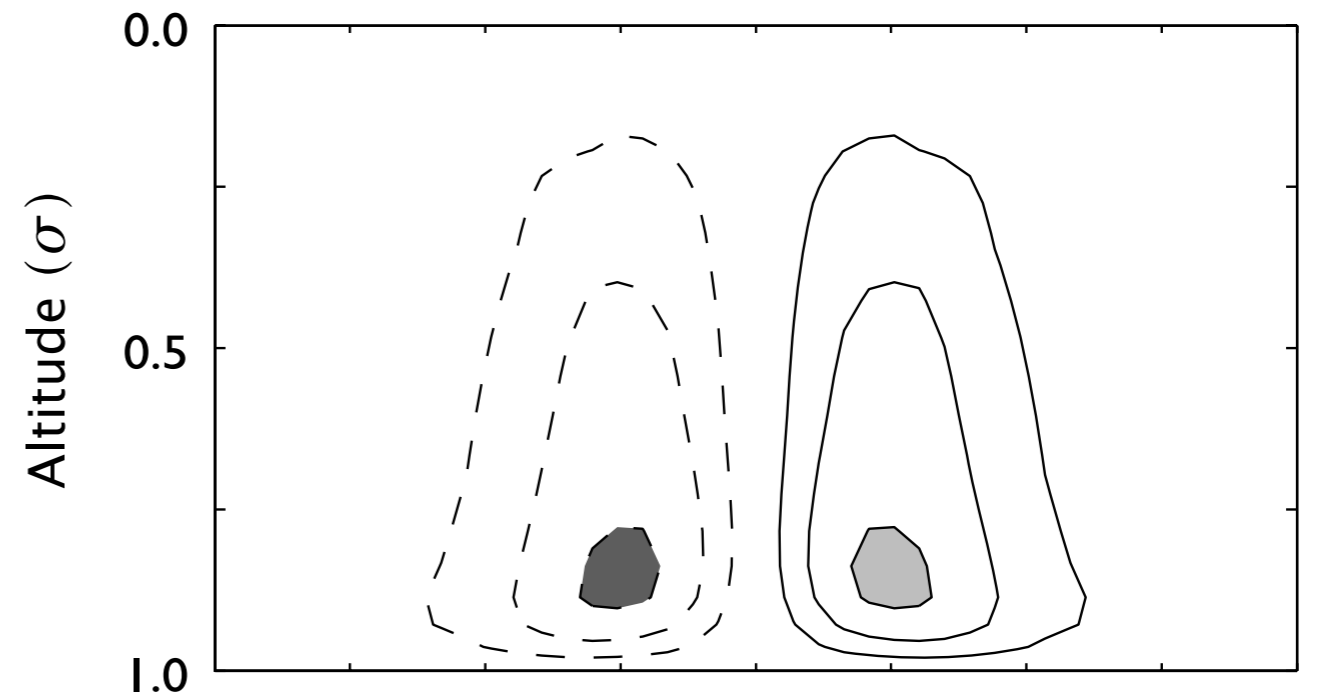
3D model, with mid-latitude baroclinic eddies.



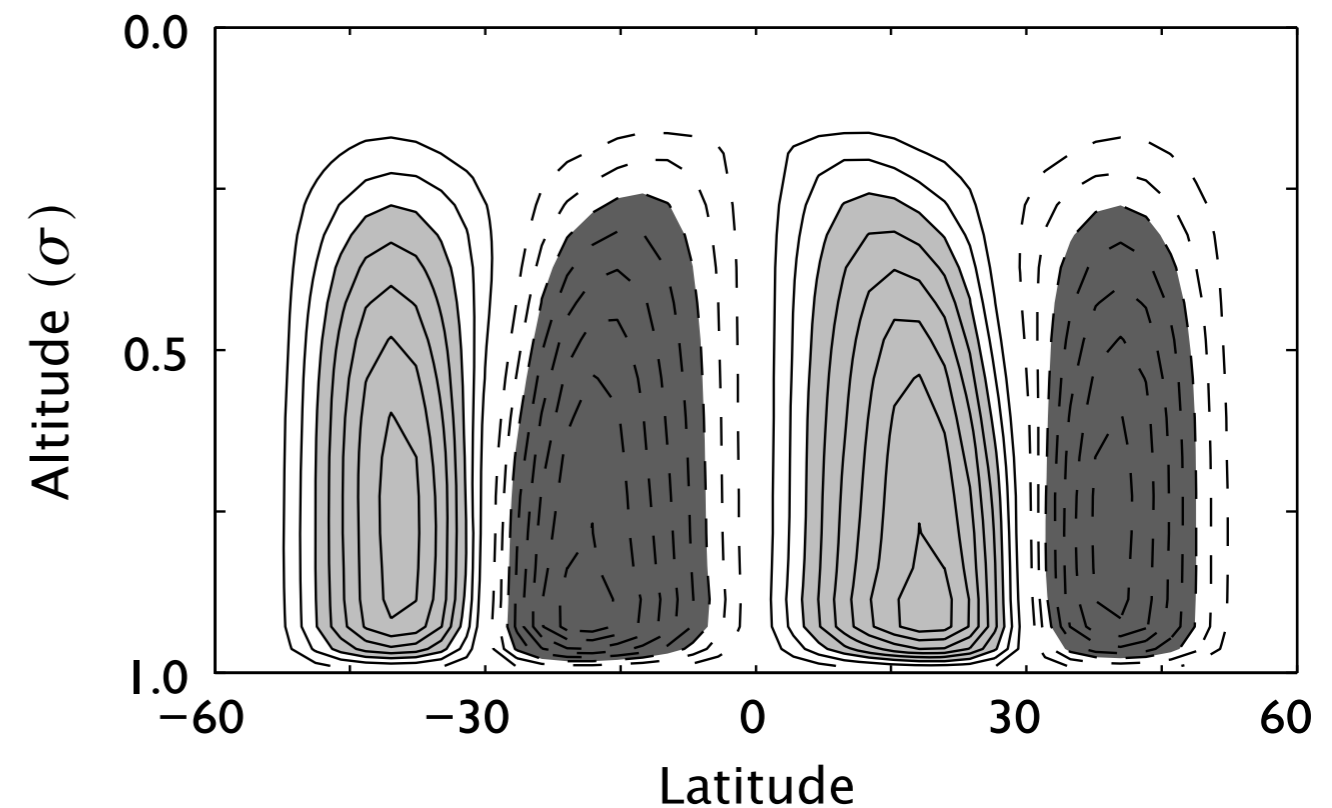
Courtesy: C. Walker

# Numerical Experiments on Hadley Cell

Axi-symmetric model, nearly angular momentum conserving. Weak Hadley Cell, no Ferrel Cell



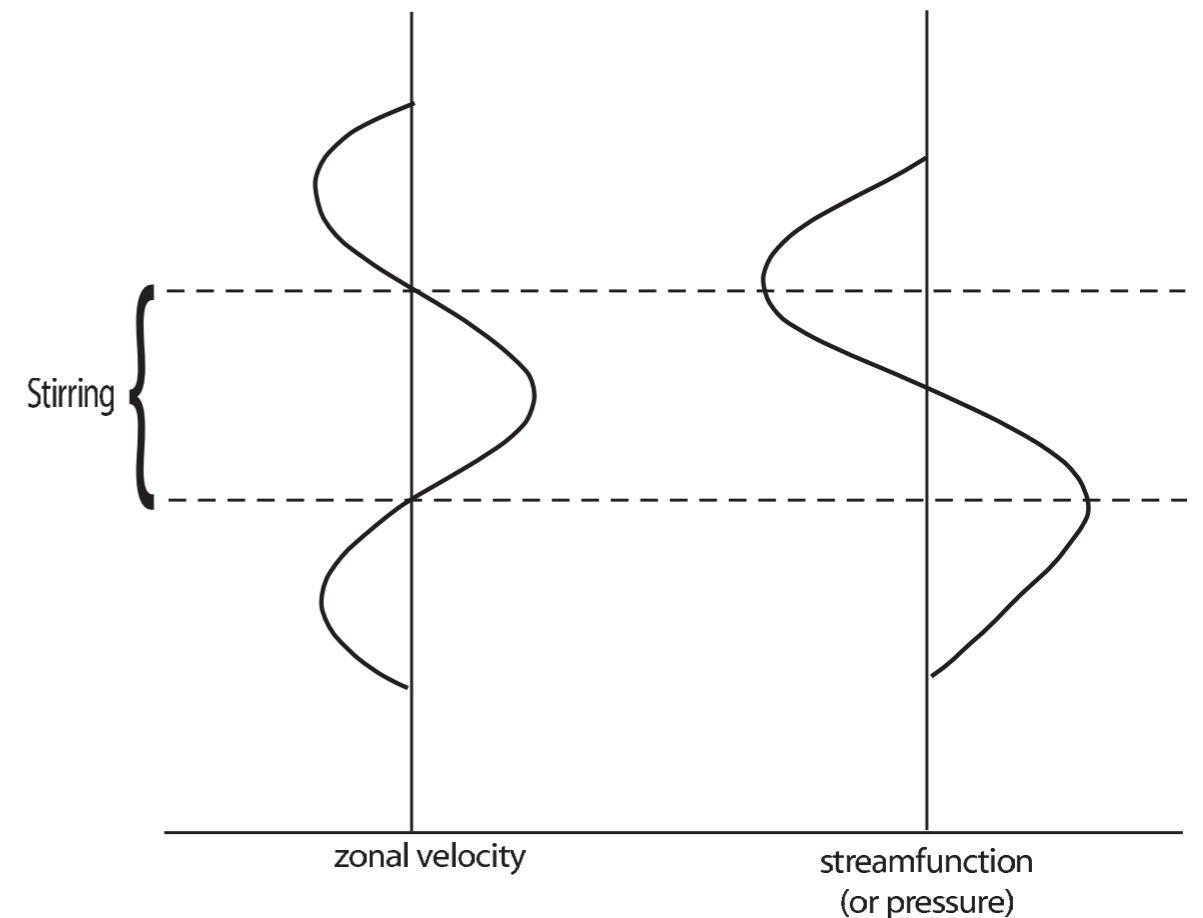
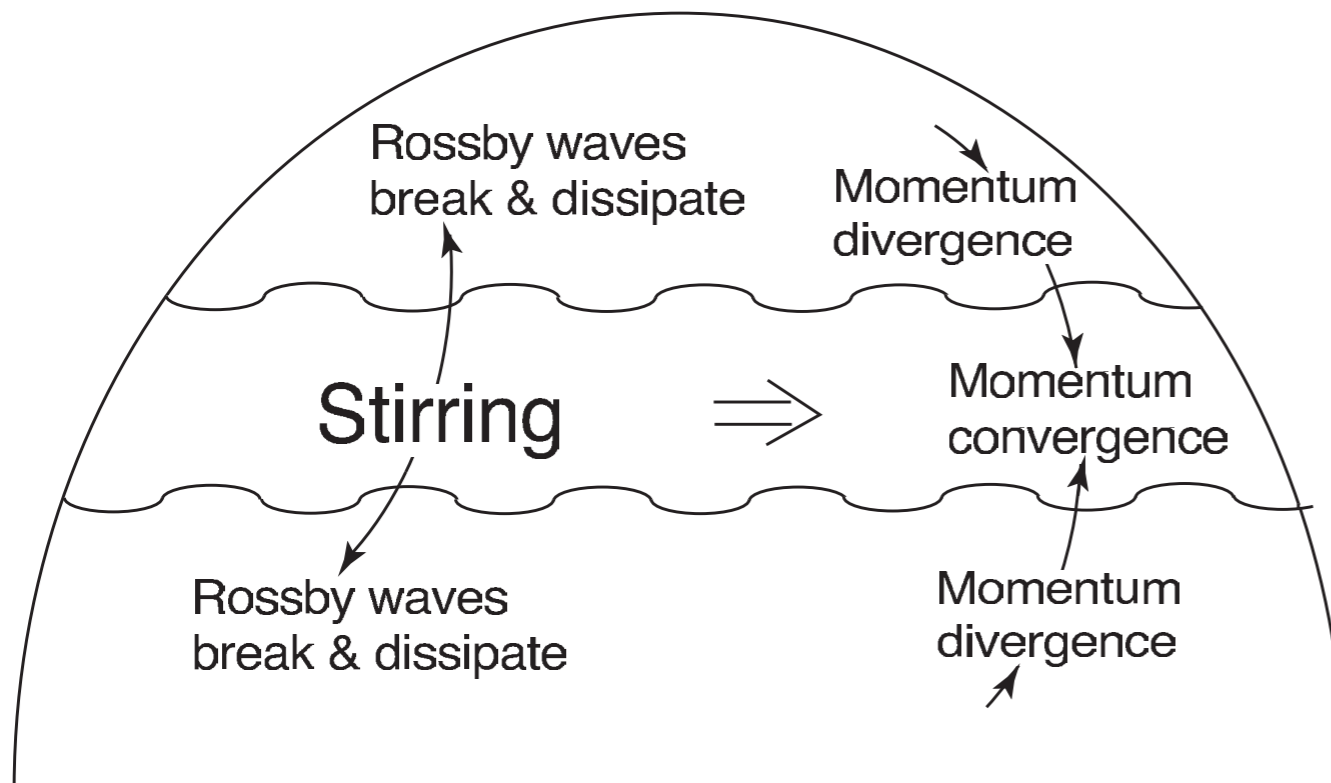
3D model, with mid-latitude baroclinic eddies. Strong Hadley and Ferrel Cells.



Courtesy: C. Walker

# Eddy-driven Jets

## Generation Mechanism in mid-latitudes



Rossby waves cause momentum convergence in the region of stirring, so generating an eastward jet.

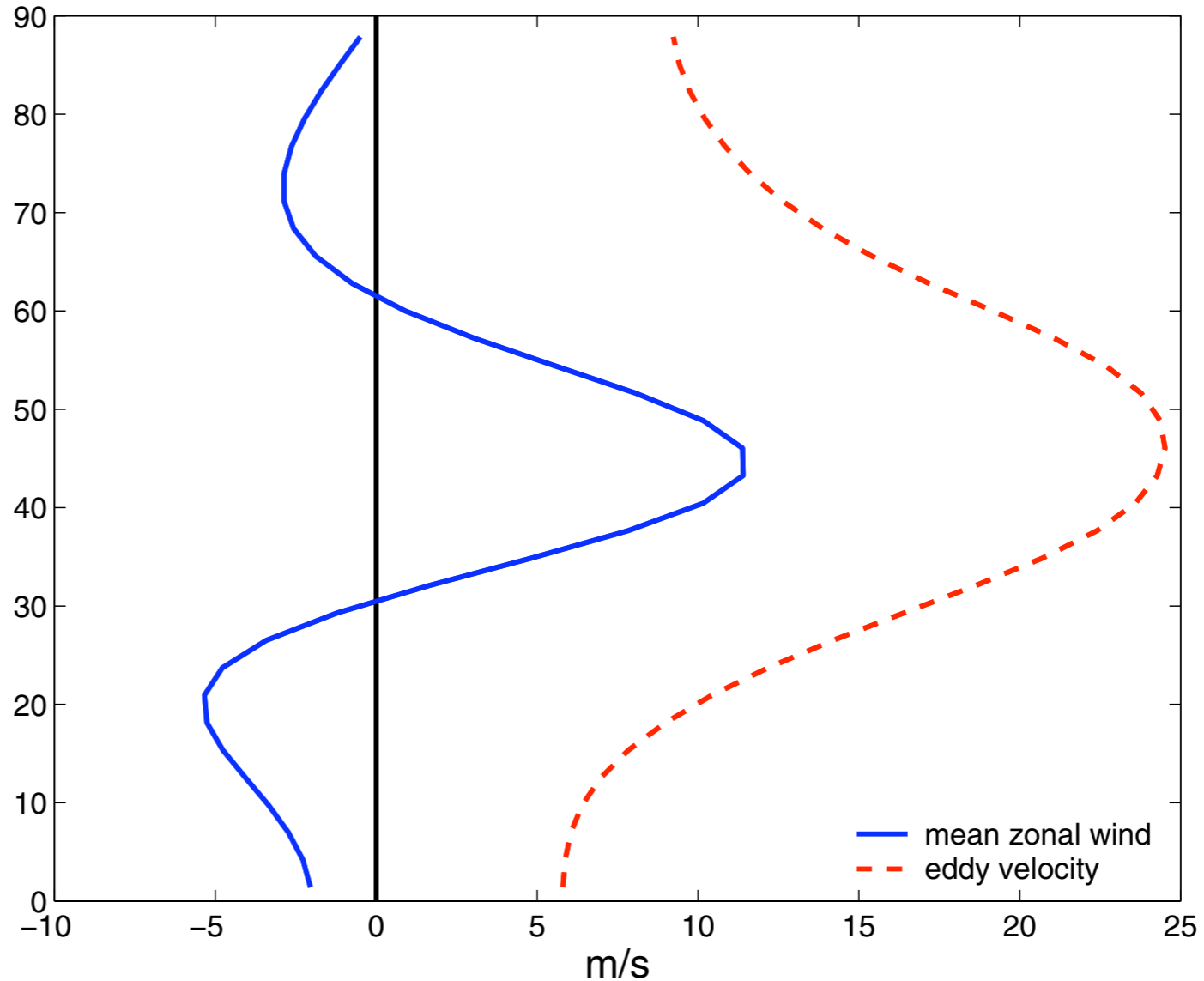
This is the most studied and probably the most robust mechanism of eastward jet generation in terrestrial atmospheres.

# Numerical Example in a Stirred Model

For experts only

$$\frac{\partial}{\partial y} \overline{u'v'} < 0$$

latitude



Solid line —  $\bar{u}$

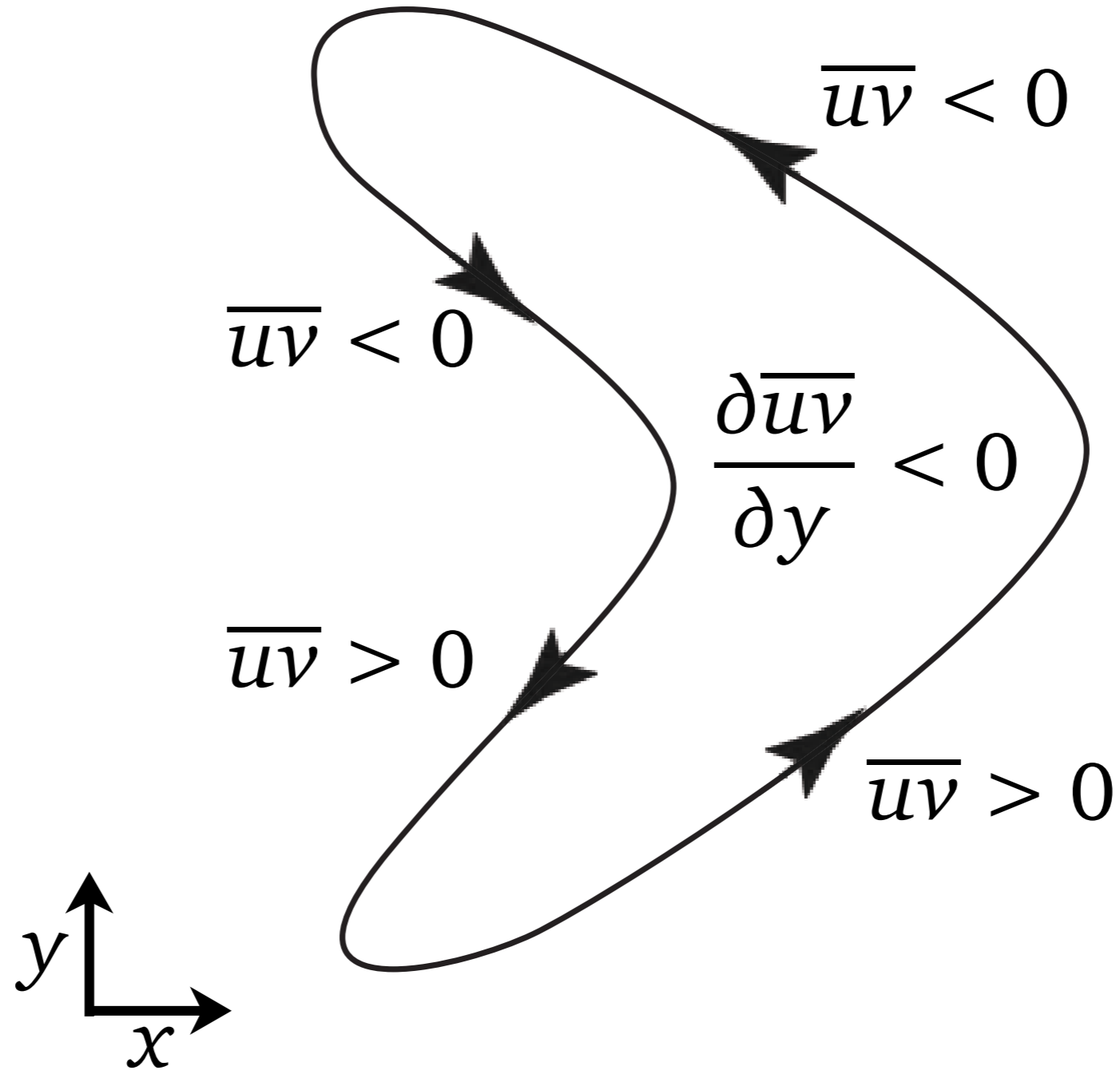
Dashed line  $\cdots$   $u_{rms}$

Equation of motion:

$$\frac{\partial \zeta}{\partial t} + J(\psi, \zeta) = S - r\zeta + \nu \nabla^4 \zeta$$



# Physical Manifestationb: Bow-shaped Eddies



## *Superrotation: What is it?*

A state in which the atmosphere rotates more quickly than the planet beneath it, such that there is a maximum of axial angular momentum in the atmospheric interior.

At the equator, angular momentum at the surface is

$$M_e = \Omega a^2$$

For a shallow atmosphere

$$M = (u + \Omega a \cos \theta) a \cos \theta$$

and if  $u = 0$ ,  $M$  is maximum at the equator. Winds are superrotating if  $M > M_e$

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- If an atmosphere superrotates, it almost certainly superrotates at the equator.
- Otherwise the angular momentum increases away from the equator, and toward the axis of rotation. This would be inertially (aka centrifugally) unstable.
- Typically, models forced randomly on the sphere do not superrotate. But some planets (Venus, Jupiter, Saturn, Titan) do superrotate.
- Has to be a mechanism to produce it, if there is diffusion (“Hide’s theorem.” )

# What is superrotation?

The condition  $M > M_e$  can be rearranged to find a lower bound for the superrotating wind,  $u_s$

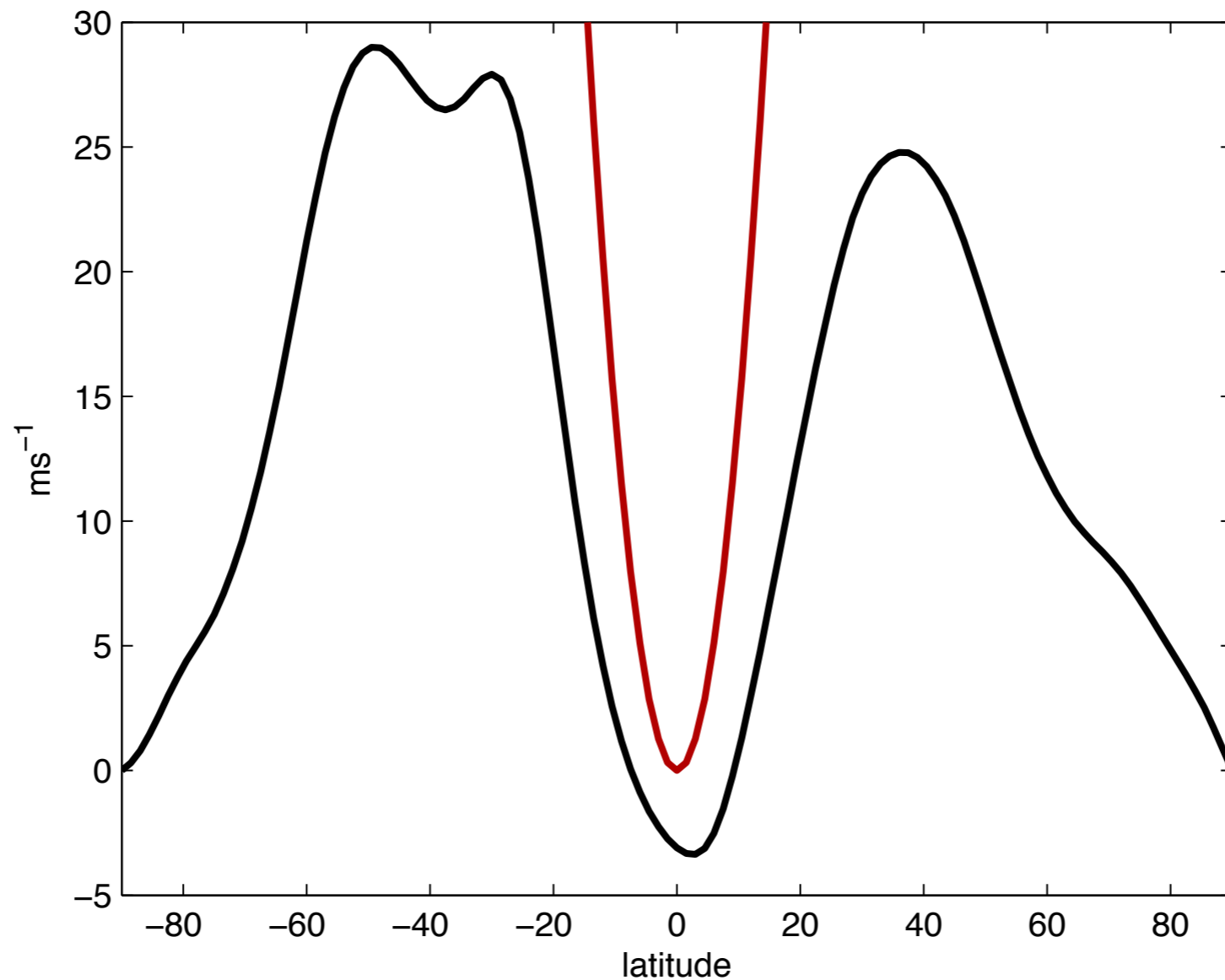
$$M > M_e$$

$$a \cos \phi (\Omega a \cos \phi + u_s) > \Omega a^2$$

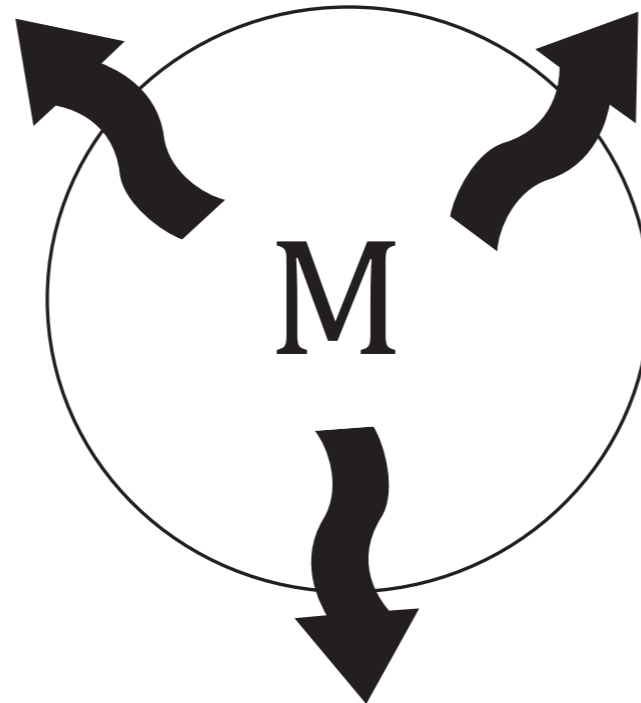
$$\rightarrow u_s > \Omega a \frac{\sin^2 \phi}{\cos \phi}$$

# *Superrotation on Earth?*

Zonal mean annual mean zonal winds at 250 hPa from ERA-Interim (black) and lower bound for superrotating winds (red). If we talk about superrotation it will almost always mean equatorial superrotation.



## *Hide's theorem*



Hide's theorem tells us that a local maximum in angular momentum (e.g. superrotation) *cannot* be maintained by the mean flow. A local maximum in angular momentum implies upgradient eddy momentum fluxes.

# Non-existence of Internal Extrema

Suppose that a quantity is advected and diffused

$$\frac{D\phi}{Dt} = \nabla \cdot \kappa \nabla \phi$$

In a steady state

$$\nabla \cdot (\mathbf{u}\phi) = \nabla \cdot \kappa \nabla \phi$$

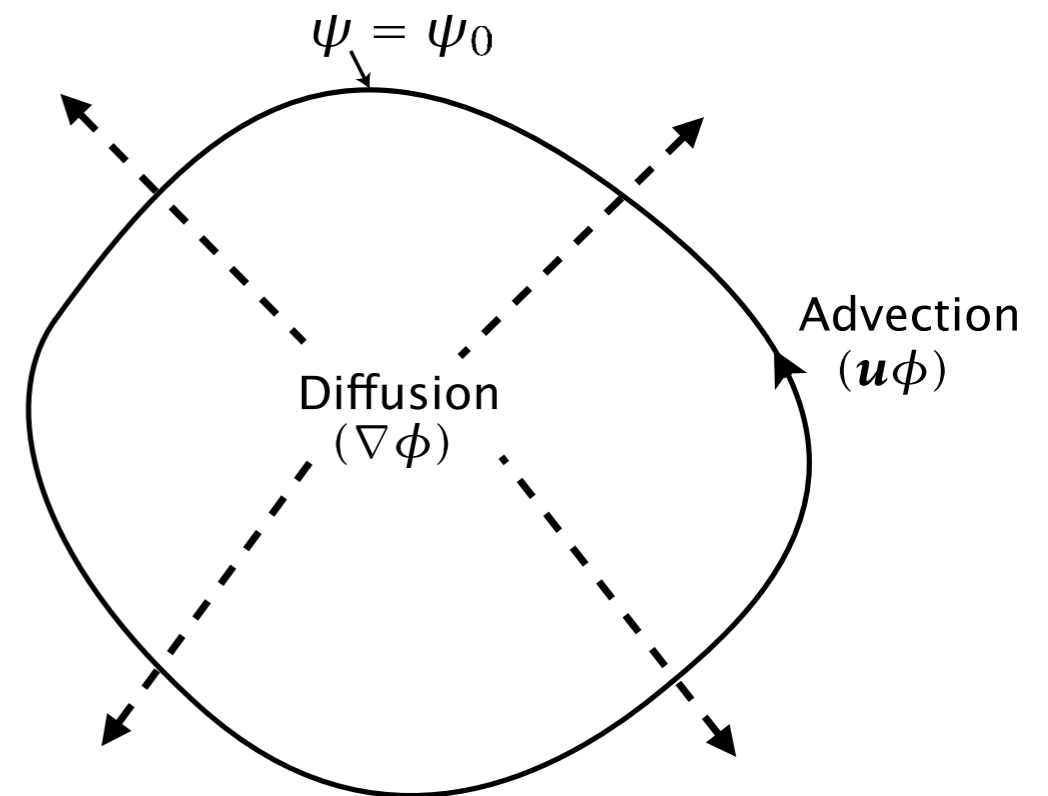
Suppose there is a maximum in the interior. Integrate over an area following an isoline of  $\phi$  surrounding the maximum:

$$\int \nabla \cdot (\mathbf{u}\phi) dA = \oint (\mathbf{u}\phi) \cdot \mathbf{n} dl = \phi \oint \mathbf{u} \cdot \mathbf{n} dl = \phi \int \nabla \cdot \mathbf{u} dA = 0.$$

But

$$\int \nabla \cdot \kappa \nabla \phi dA = \oint \kappa \nabla \phi \cdot \mathbf{n} dl \neq 0$$

Diffusion will dissipate the extremum, and advection can do nothing to replace it.



# Angular Momentum

## Hide's 'Theorem'

Angular momentum is not quite an advective-diffusive quantity, because momentum is diffused, not angular momentum. For zonal averages

$$\frac{Dm}{Dt} = \nu_v \frac{\partial^2 m}{\partial z^2} + \nu_h \frac{\partial^2 u}{\partial y^2}.$$

Ergo:

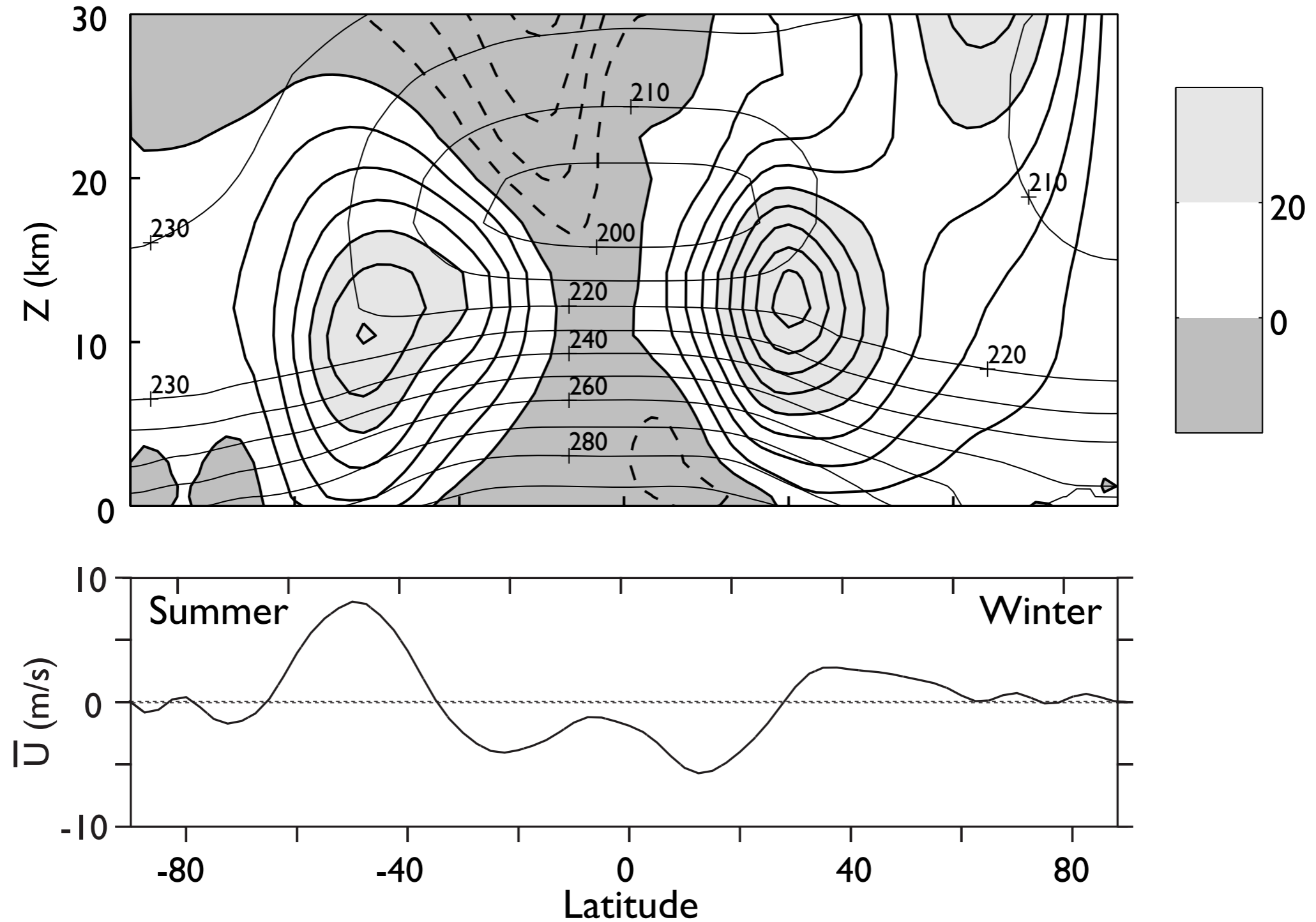
- 1 Superrotation may be diffused away by vertical diffusion.
- 2 The horizontal component will try to create solid body rotation, which may be superrotating. This is the so-called Geirasch mechanism.



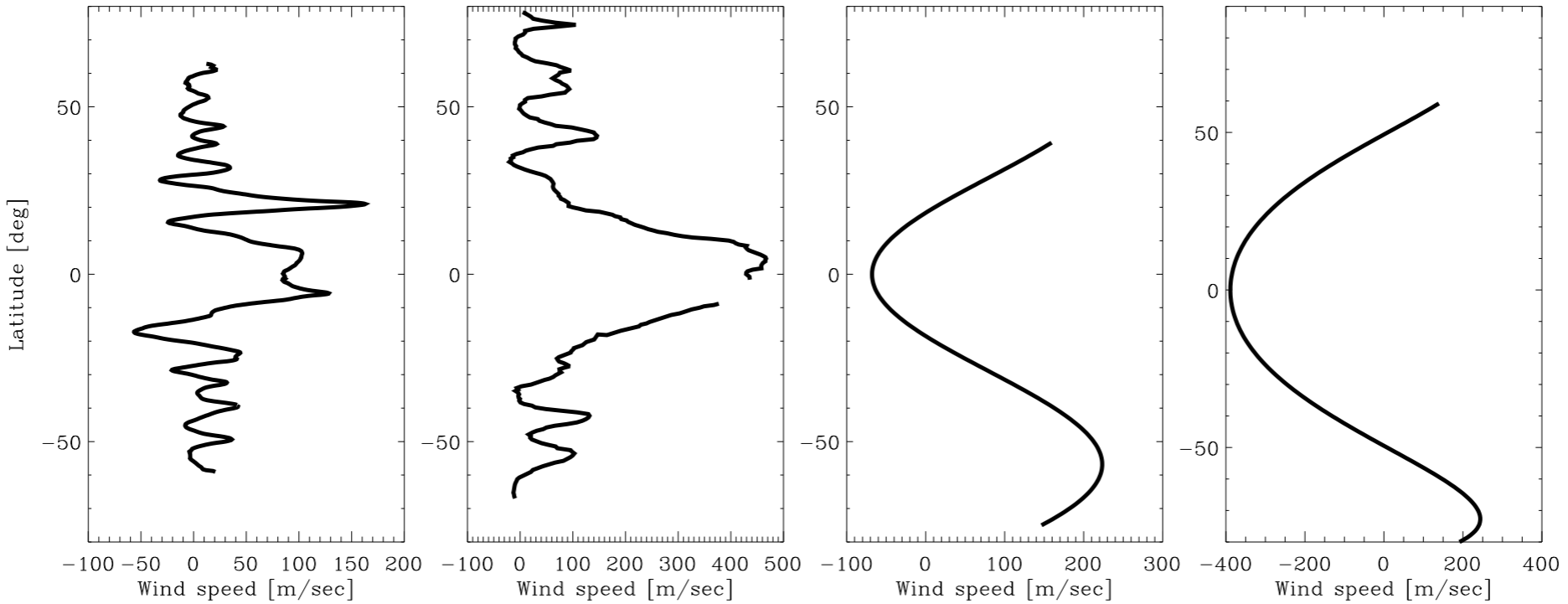
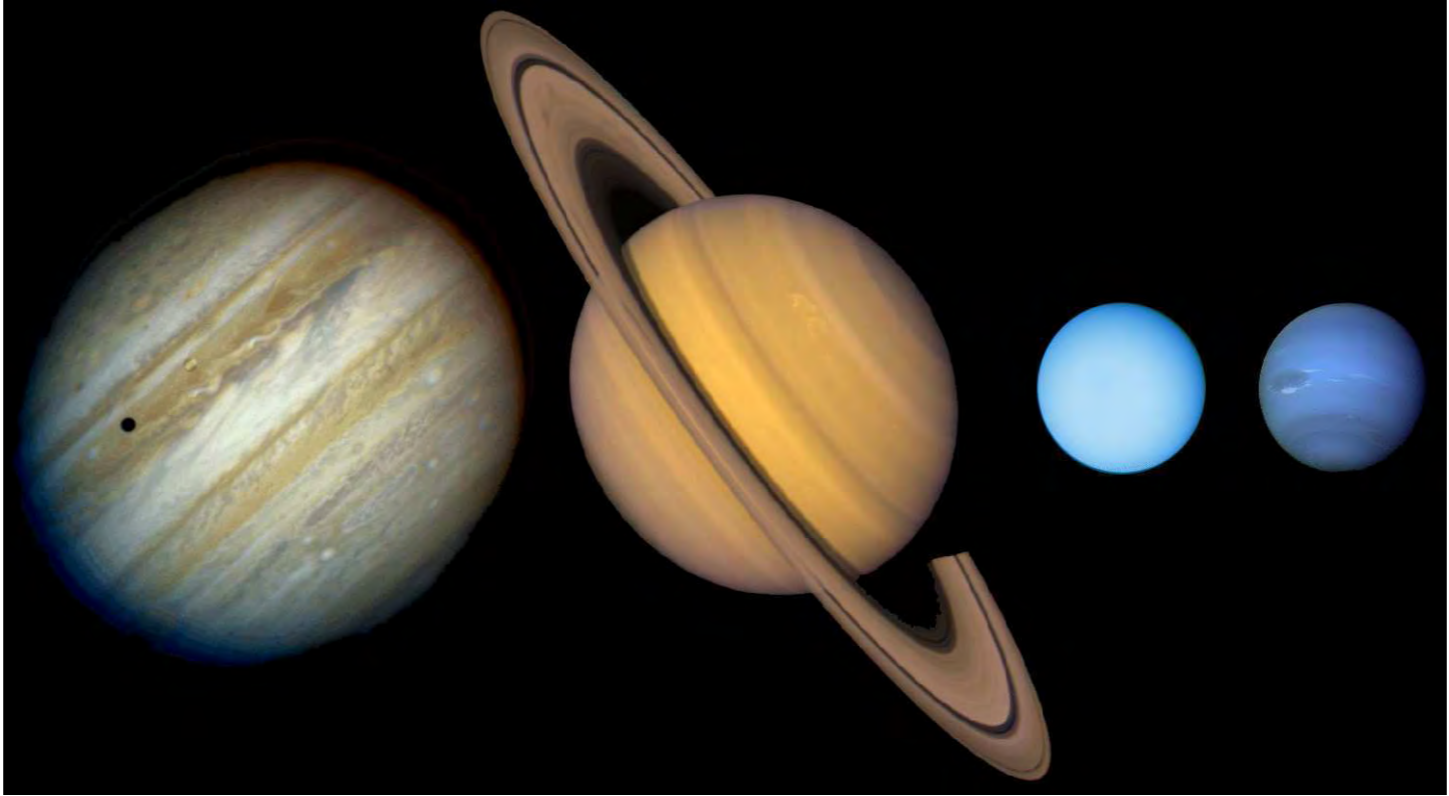
# Observations

## Earth (DJF)

Easterlies all the way up at the equator

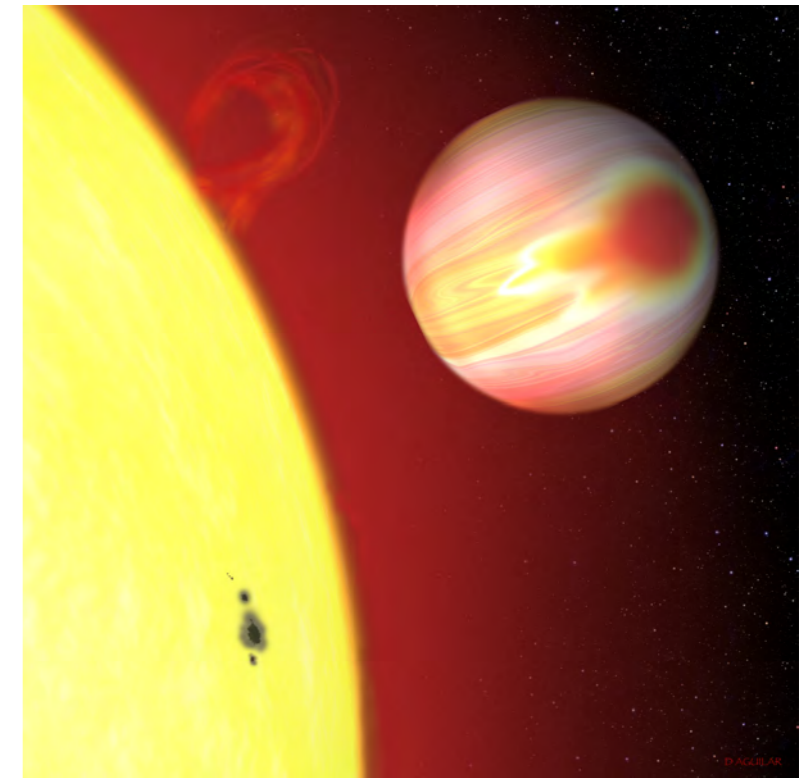


# Observations (Giant Planets)



# Mechanisms of Superrotation

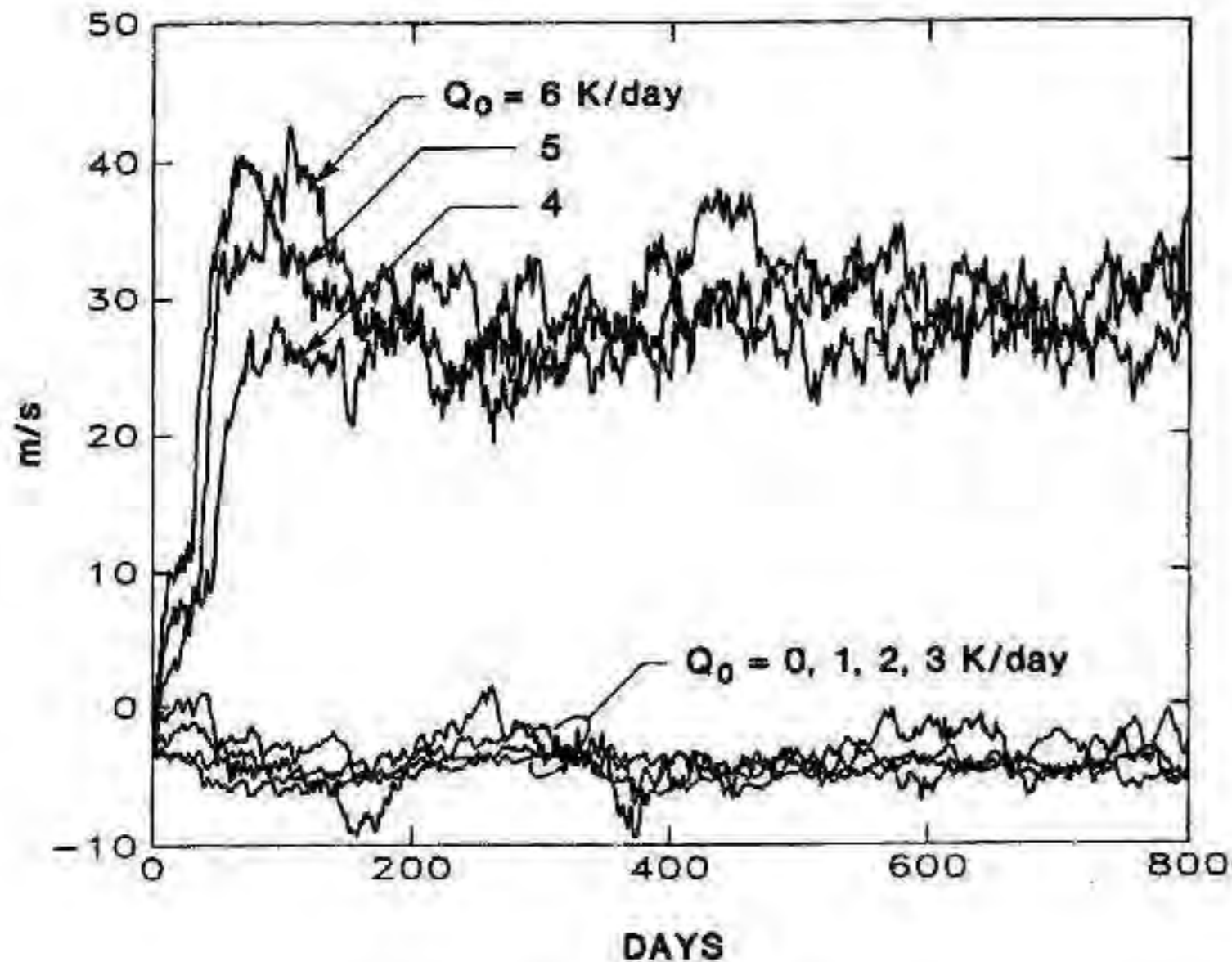
- Forcing at the equator. Convection or stationary waves (Many authors: Suarez & Duffy, Saravanan, Showman and collaborators.)
  - ▶ Excites Rossby waves at equator, EP flux propagates away, converging momentum at equator.
  - ▶ Convection → Rossby waves is plausible for Jupiter and Saturn. Straightforward and robust.
- So-called ‘Geirasch mechanism’
  - ▶ A little ill-defined (IMHO)
  - ▶ Barotropic instability (off equatorial?) creates an eddy-diffusivity that has same form as a horizontal molecular diffusion.
  - ▶ Evolves into solid body rotation, which is superrotating.
  - ▶ But if the barotropic instability is off-equator, it seems that Rossby waves would be excited there, converging momentum off the equator.
- Mechanisms involving gravity waves?
  - ▶ Vertical propagation of momentum by gravity waves.
  - ▶ QBO, moving flame. (G. Schubert, Whitehead, Lindzen-Holton-Plumb)



# Model Results of Others

## Earth like

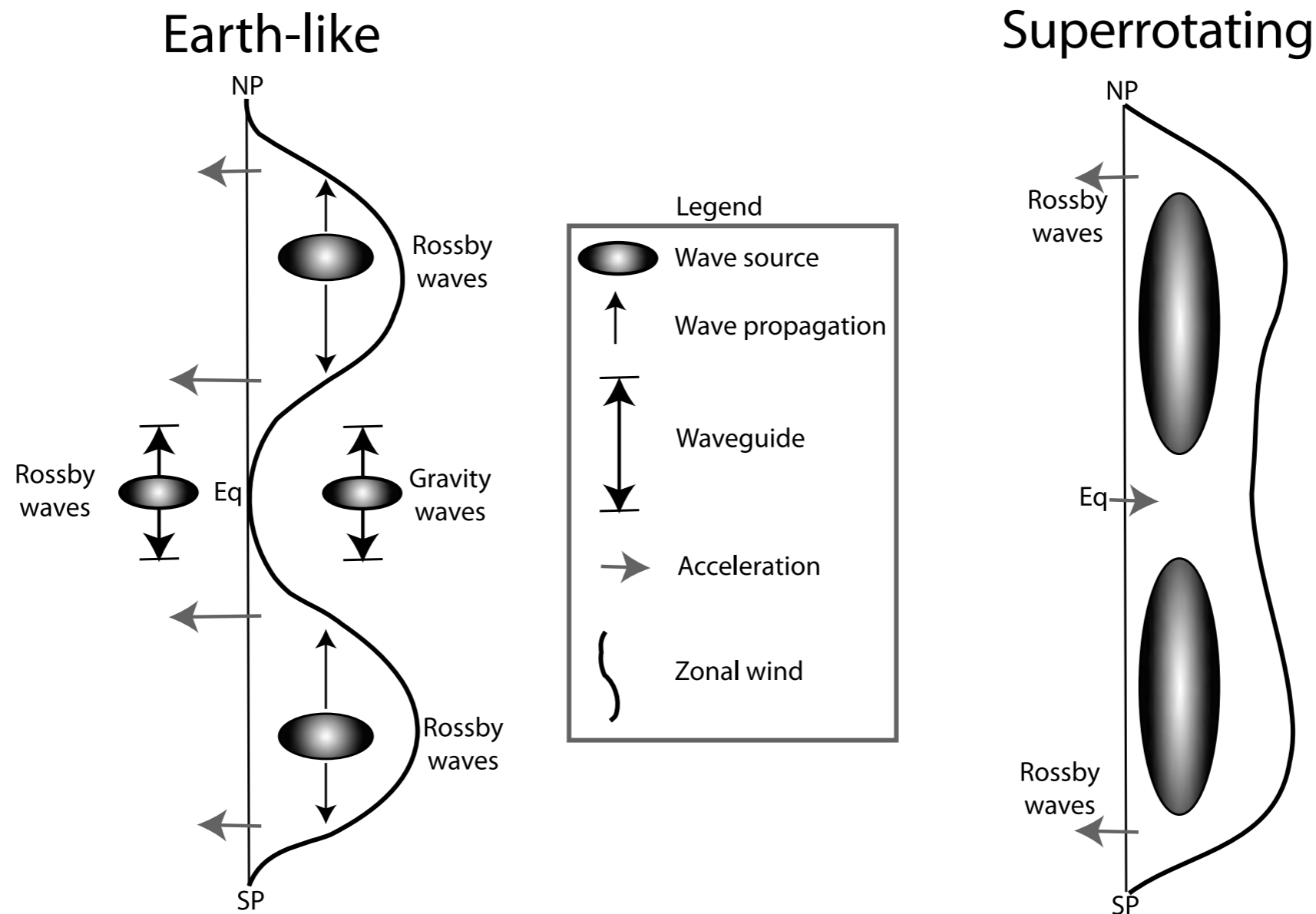
Stationary wave forcing produces a transition to superrotation in an Earth-like regime.  
(Suarez and Duffy, Saravanan)



- Add asymmetric heating. Sudden bifurcation seems to occur.
- Rossby waves propagate away from tropics (pseudomomentum source). Momentum converges at equator, making superrotation.
- Why is the effect so abrupt?

# *Superrotation: multiple equilibrium and hysteresis?*

There is a lack of critical latitudes in the superrotating case (Held 2000). Rossby waves emanating from midlatitudes will not decelerate the tropics and the circulation may jump into a superrotating state.



# Spontaneous Superrotation

- Original goal to look at Titan. But it became a GFD problem.
- Titan is a ‘terrestrial’ body: it has a rocky surface. More like Earth, and not like a giant planet.
- Spectral, GFDL dynamical core.
- Dry, Newtonian relaxation of temperature (Held–Suarez like).  
 $D\theta/Dt = -(\theta - \theta^*(y, z))/\tau$ .
- Able to vary rotation rate, size of planet, friction, etc.
- No seasonal cycle in today’s results.

Three main nondimensional parameters:

Relaxation rate of thermodynamics relative to rotation rate:

$$\hat{\tau} = 2\Omega\tau ,$$

Ekman number:

$$E = \frac{\nu}{2\Omega a} ,$$

Thermal Rossby number (Rossby number based on thermal wind as given by forcing)

$$Ro = \frac{U}{2\Omega a} = \frac{RT_0\Delta_H}{(2\Omega a)^2}$$

where  $\Delta_H$  gives the pole–equator gradient of forcing temperature.

# Experiments

*Change only the thermal Rossby number*

(Also Williams, Luz, del Genio, Yamamamoto, others.)

Do this by changing the size of the planet, similar to changing the rotation rate.

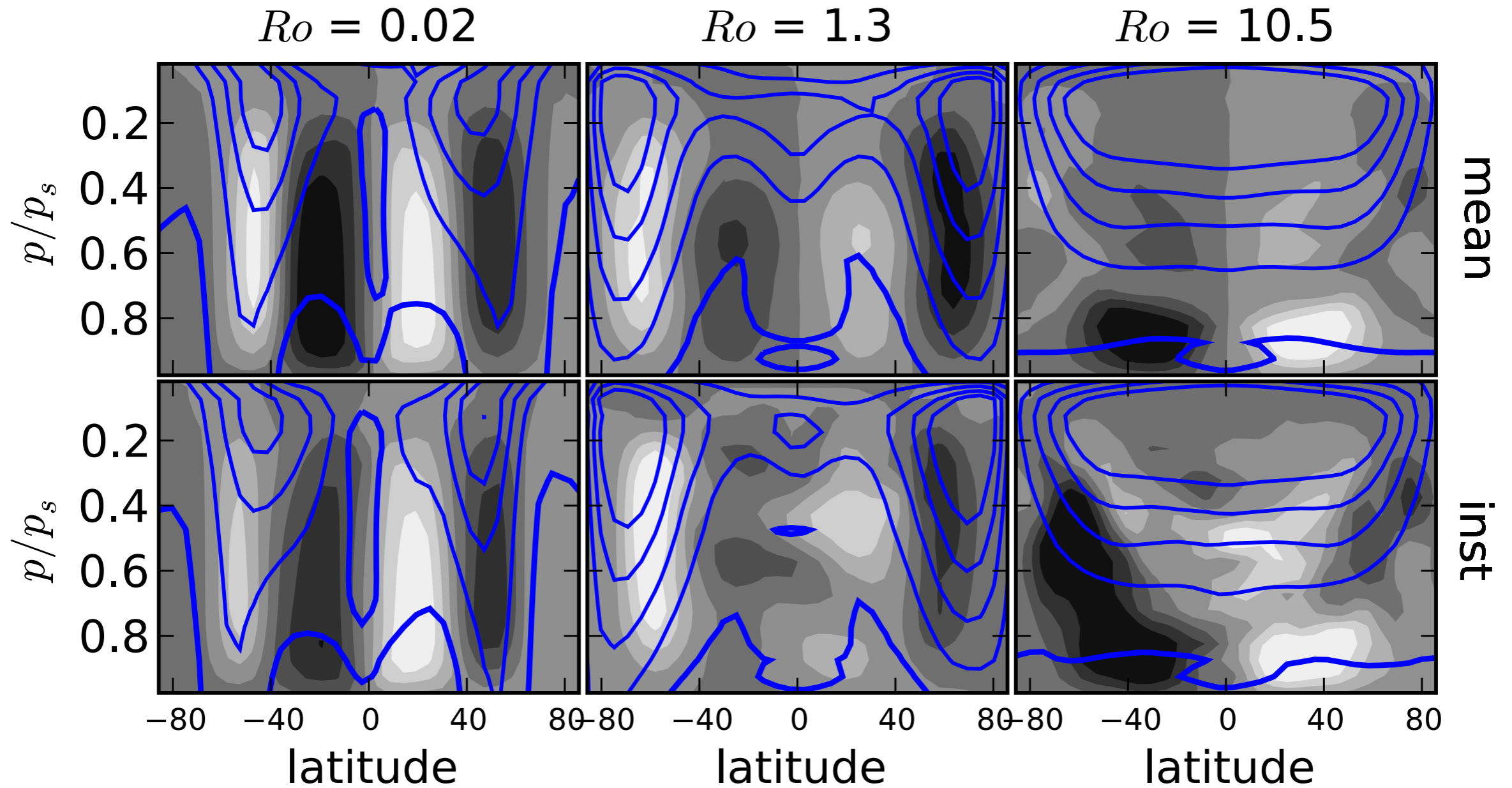
## Expectations

- 1 As we reduce rotation rate, deformation radius becomes bigger. Eventually baroclinic instability will cease.
- 2 Hadley cell more angular momentum conserving. More inertial and perhaps unsteady (symmetric instability).
- 3 More horizontal shear, possibilities for tropical instabilities.
- 4 Seasonal cycle, if present, will temper the superrotation.

# Results

## Zonal Wind and MOC

$Ro$  changes



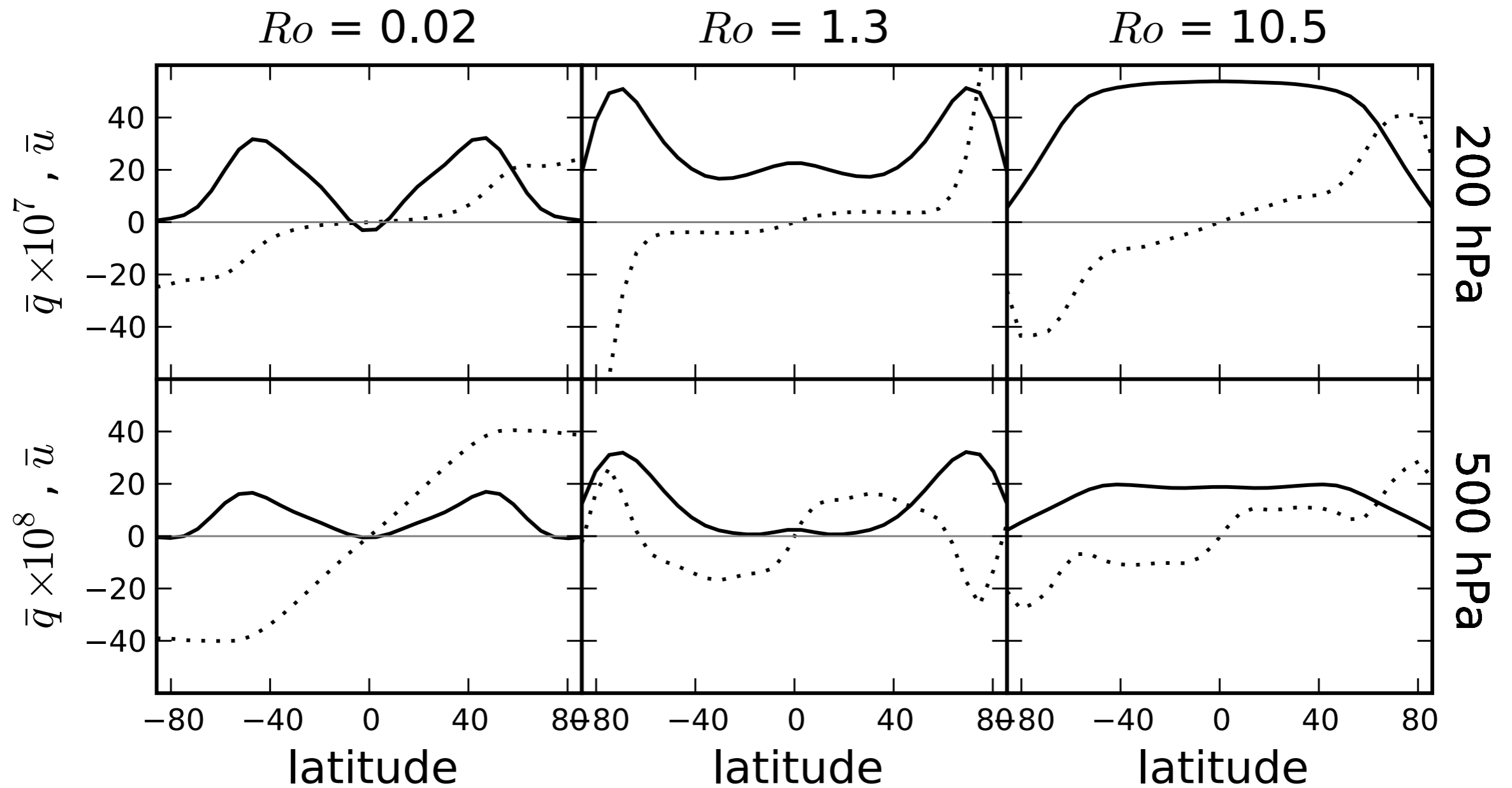
Superrotation appears as  $Ro$  increases.



# Results

Zonal Wind (solid) and PV (dotted)

$Ro$  changes



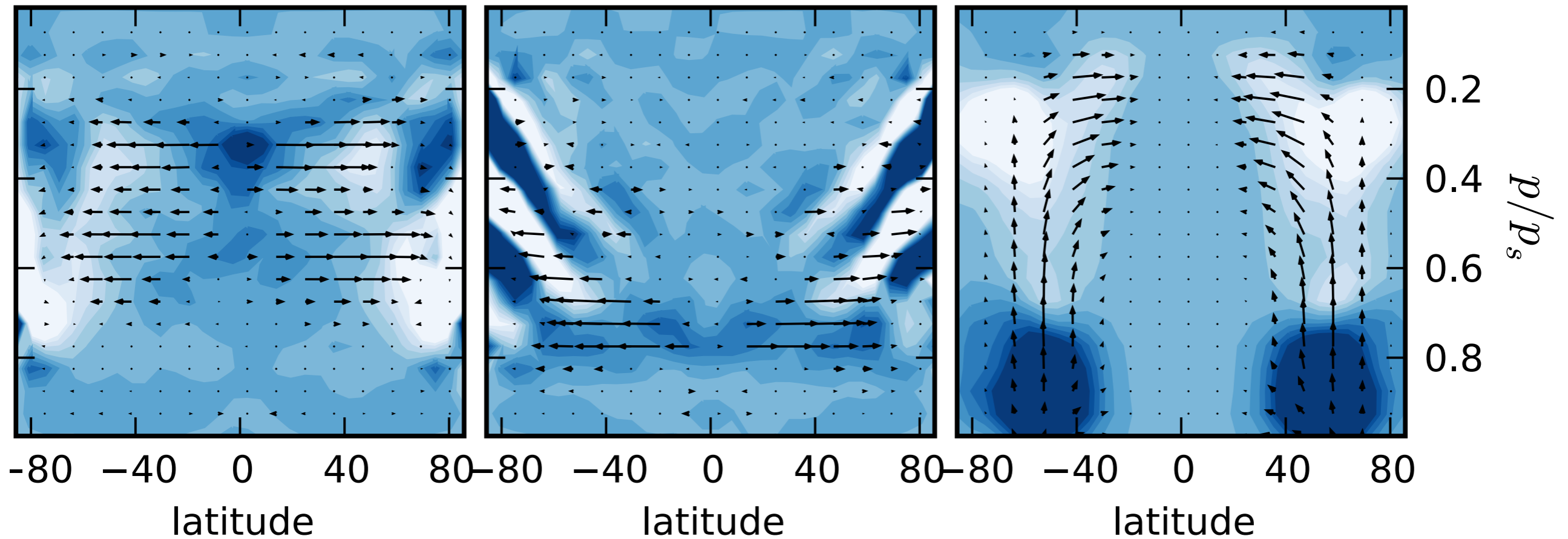
# Results

## EP fluxes and their divergences

$Ro=10.5$ , spinup

$Ro=10.5$ , steady

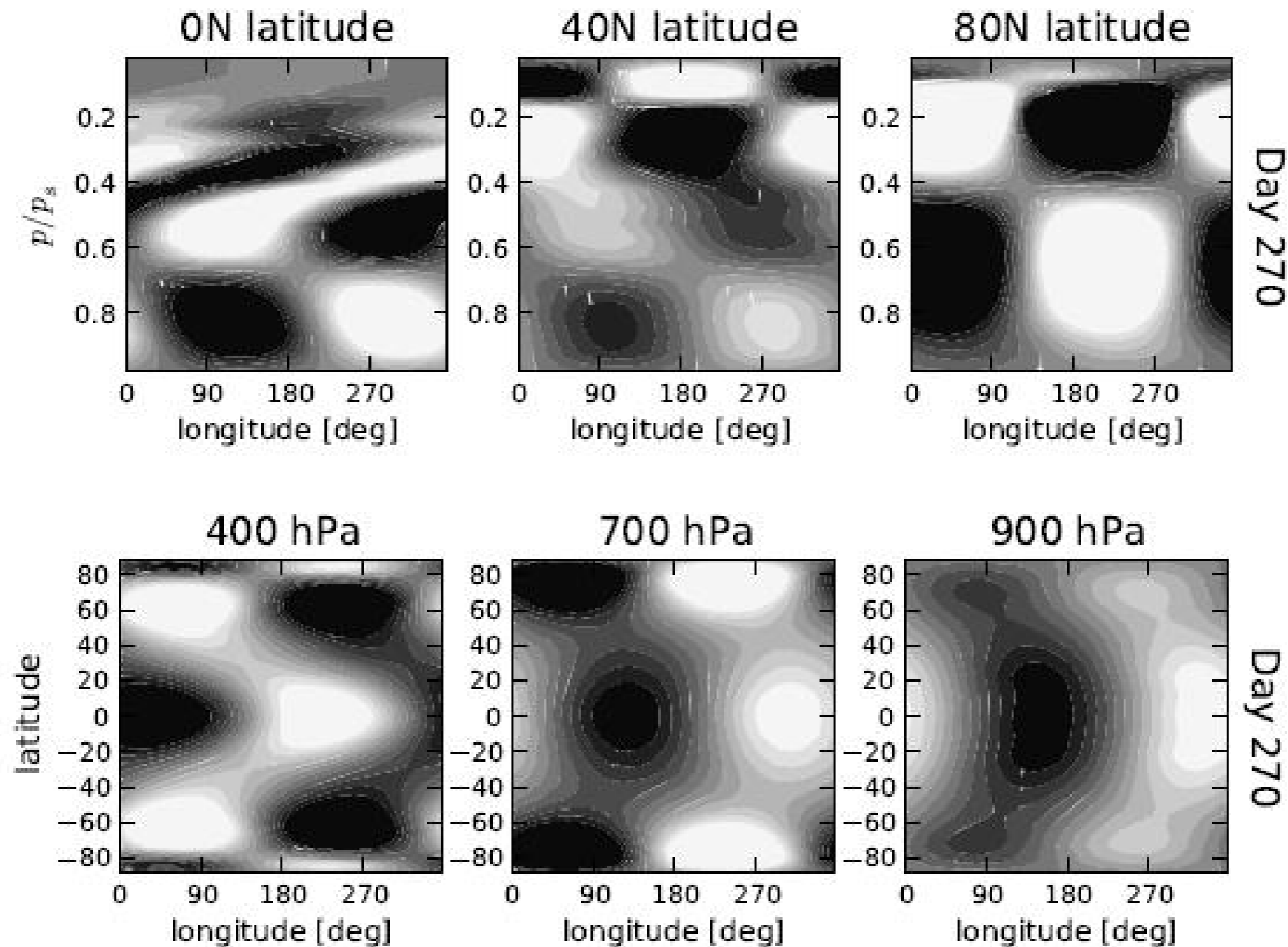
$Ro=0.02$ , steady



Looks like waves emanating from equator, but direct evidence is weak.

# Changing planetary geometry

As the thermal Rossby number increases the planetary circulation comes to be dominated by a global wavenumber-1 mode.



**Figure:** Mitchell and Vallis 2010. Geopotential height fields during spinup filtered for the dominant mode of variability.

# Thermal Rossby Number

Thermal Rossby number has a dramatic impact on equatorial winds.

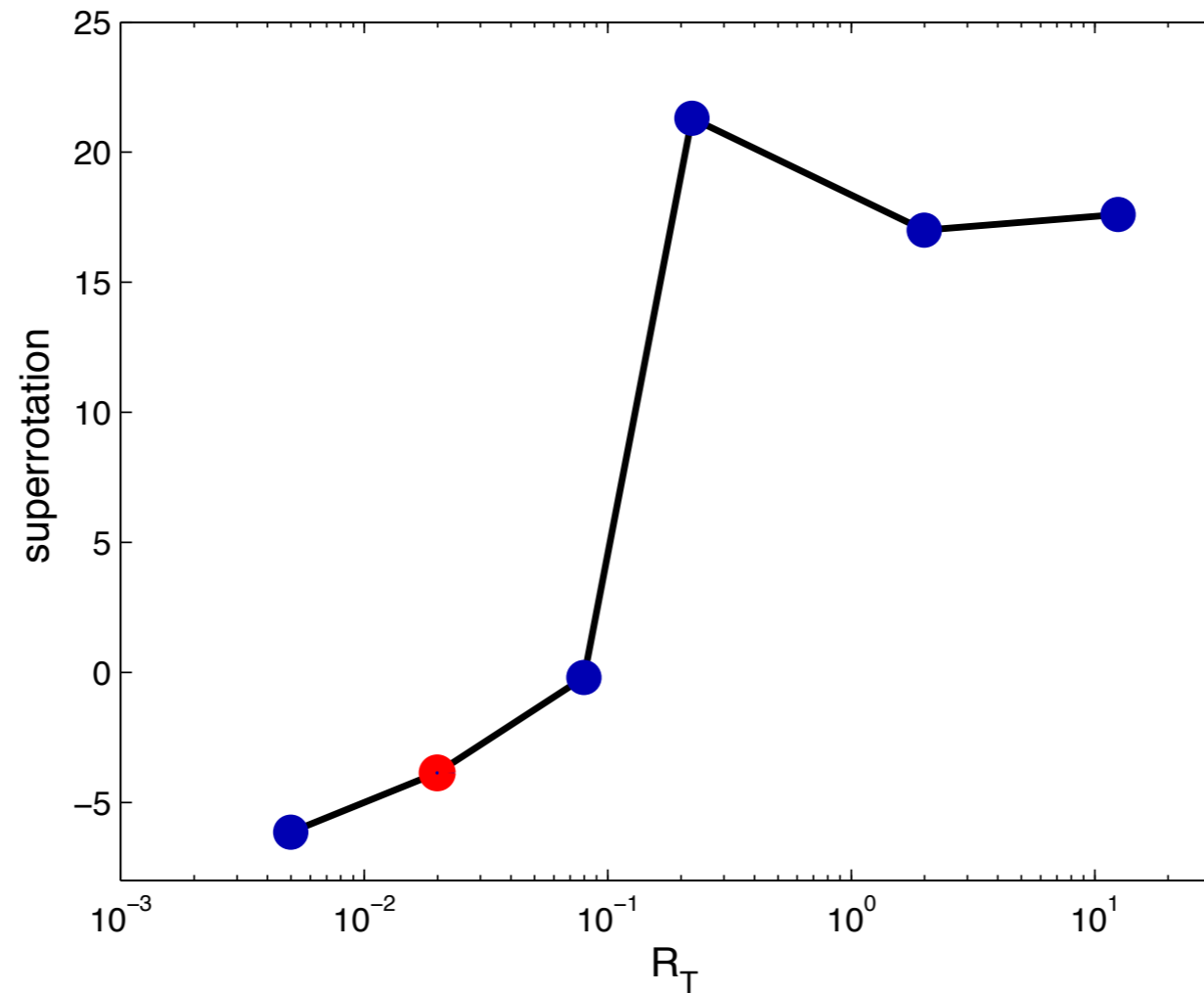


Figure: Maximum equatorial winds as a function of thermal Rossby number. Control denoted by red circle.

# Ekman Number

The Ekman number is able to generate strong easterlies, but does not create significant superrotation.

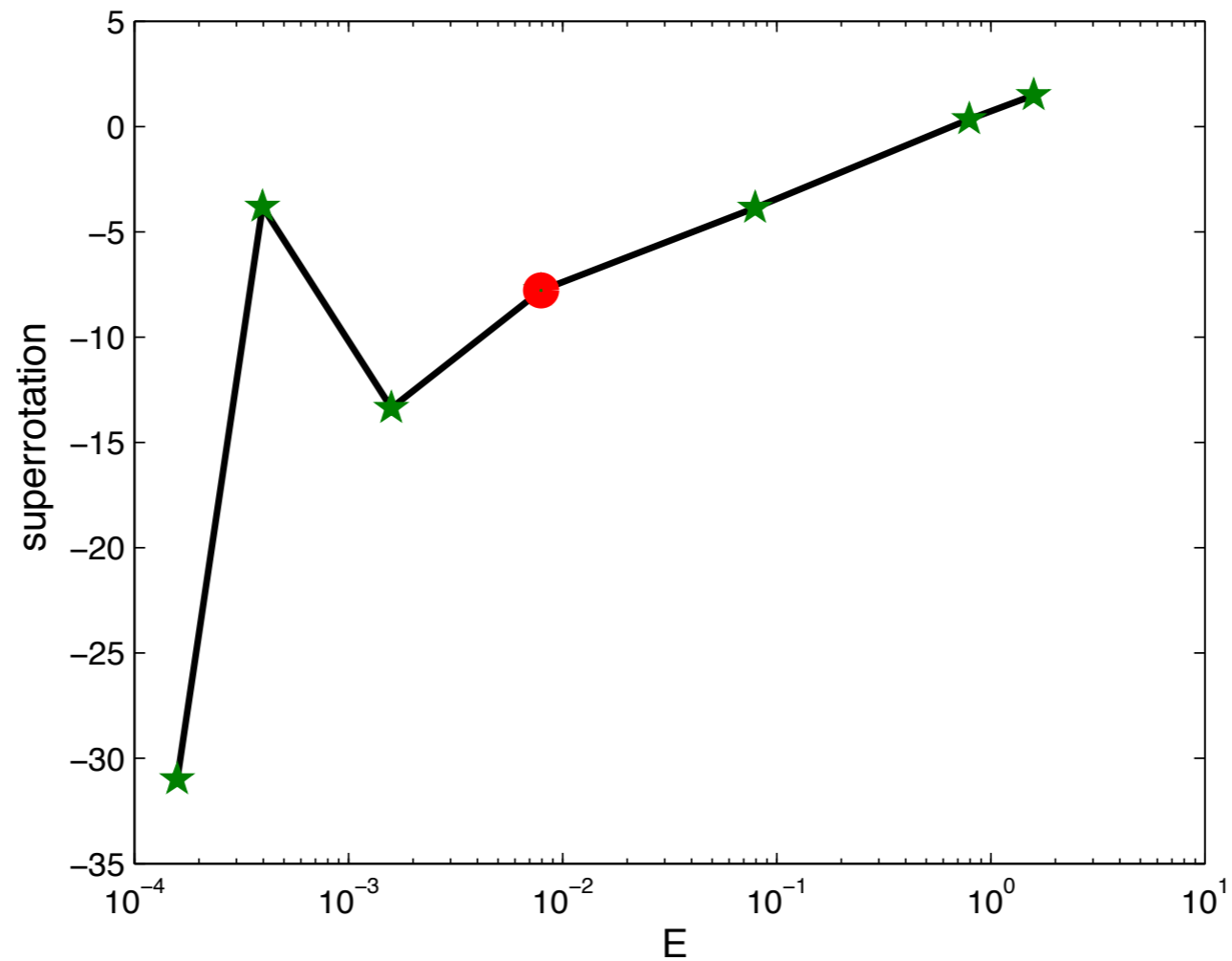
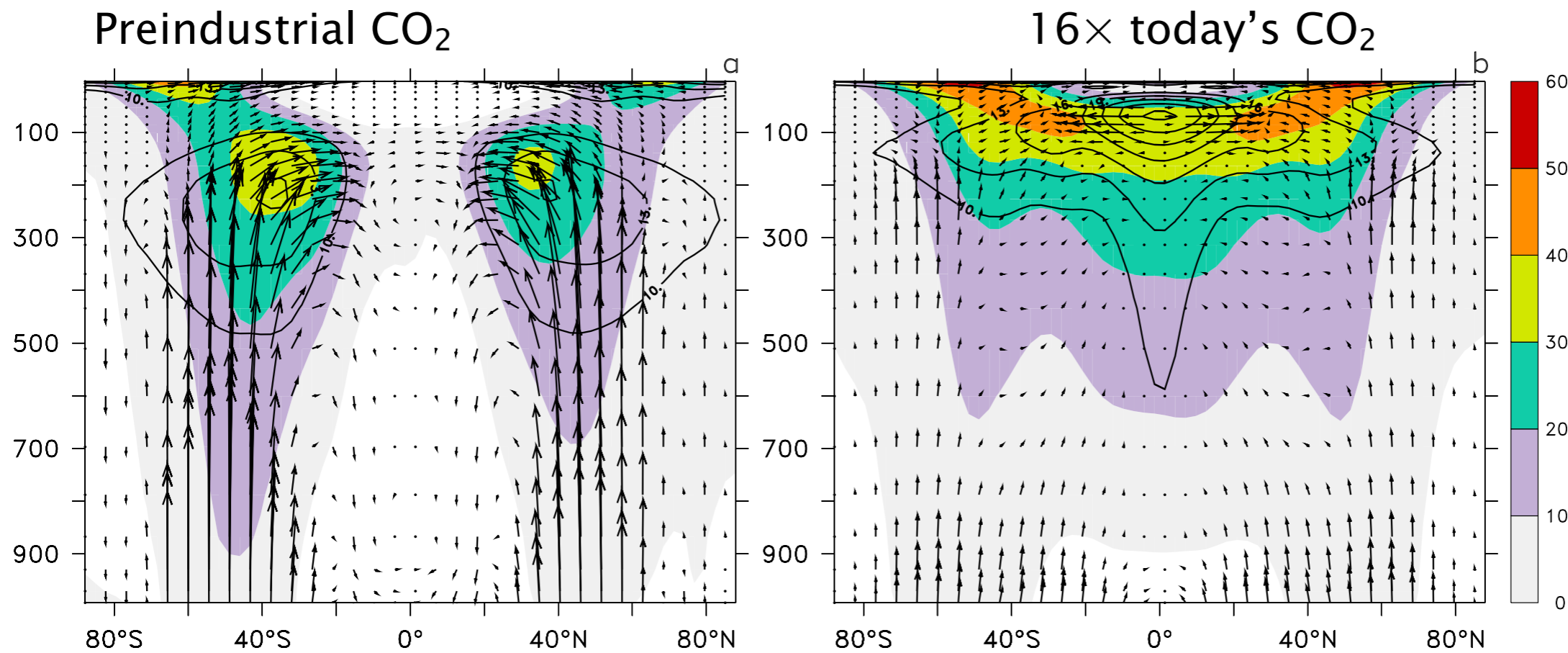


Figure: Maximum equatorial winds as a function of Ekman number. Control denoted by red circle.

# Superrotation in a hothouse

Caballero and Huber 2010 looked at the atmospheric circulations of hothouse paleoclimates using a comprehensive AGCM. With sufficiently hot temperatures the atmosphere superrotates!



Caballero and Huber 2010. Zonal mean zonal winds (coloring), standard deviation of winds (contours), and Eliassen-Palm flux (arrows). 280 ppm (left) to 8960 ppm (right)

# Superrotation in a hothouse

The superrotation is driven by a wavenumber-1 equatorial wave that resembles the Madden-Julian Oscillation.

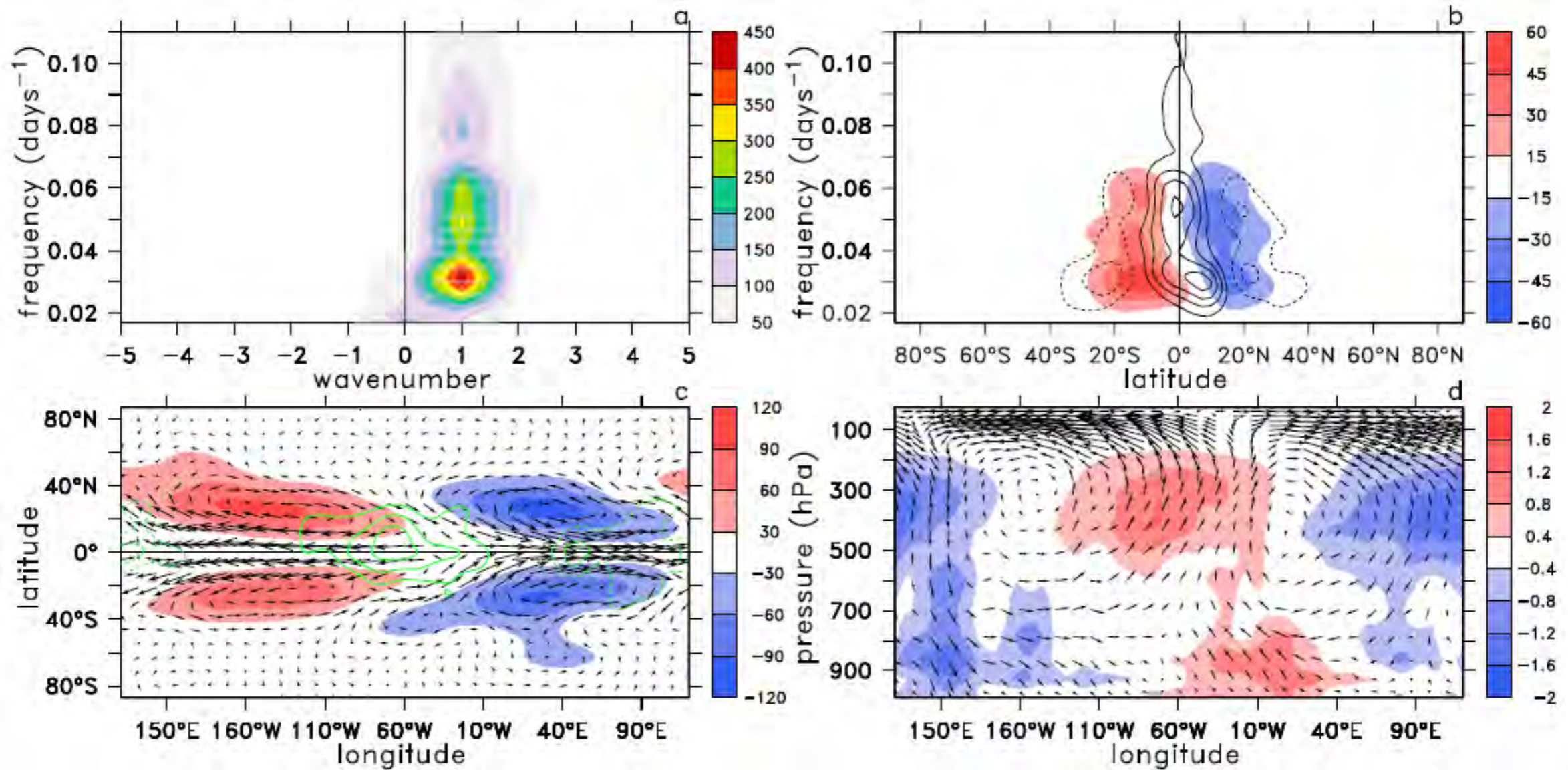
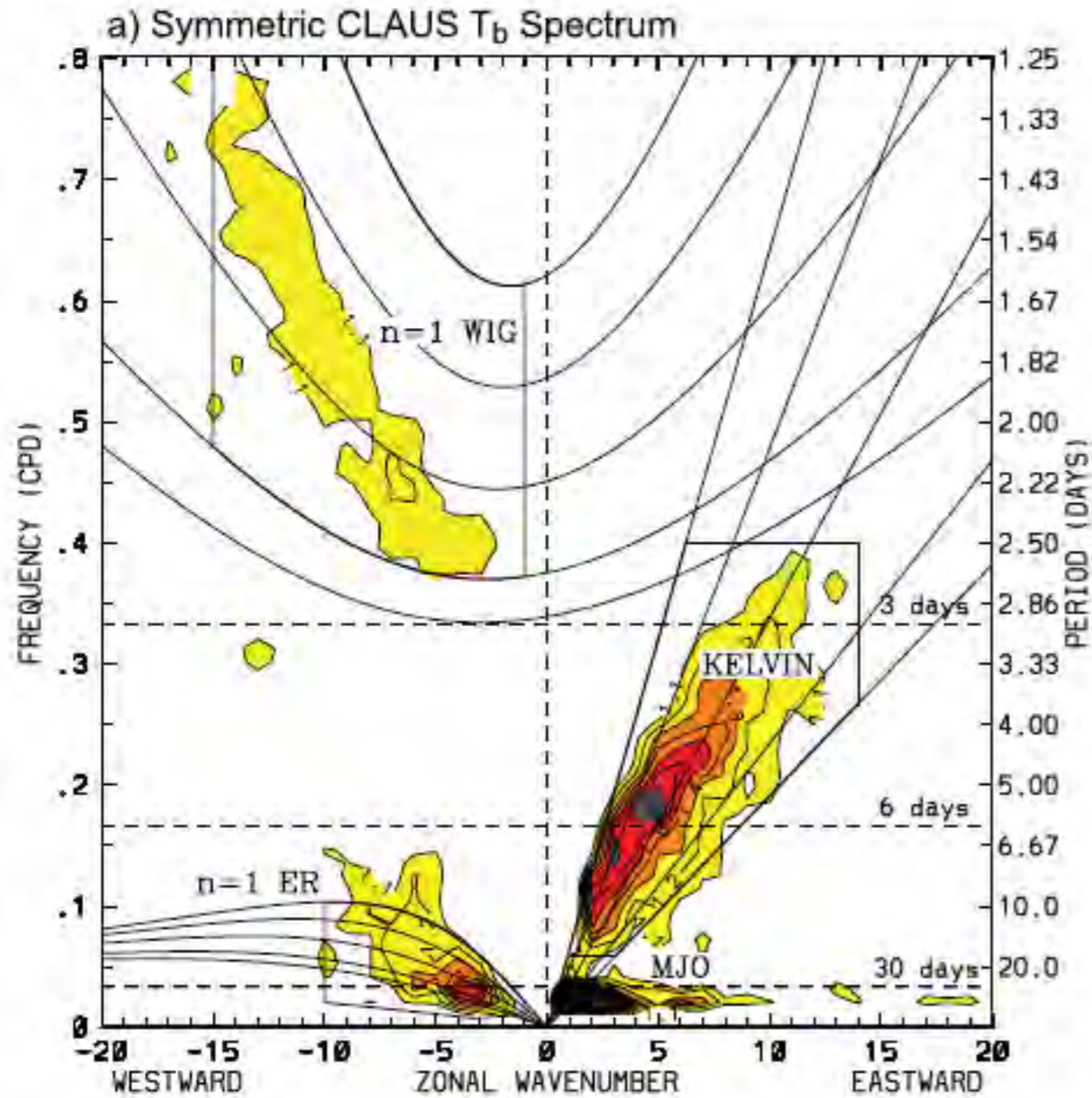


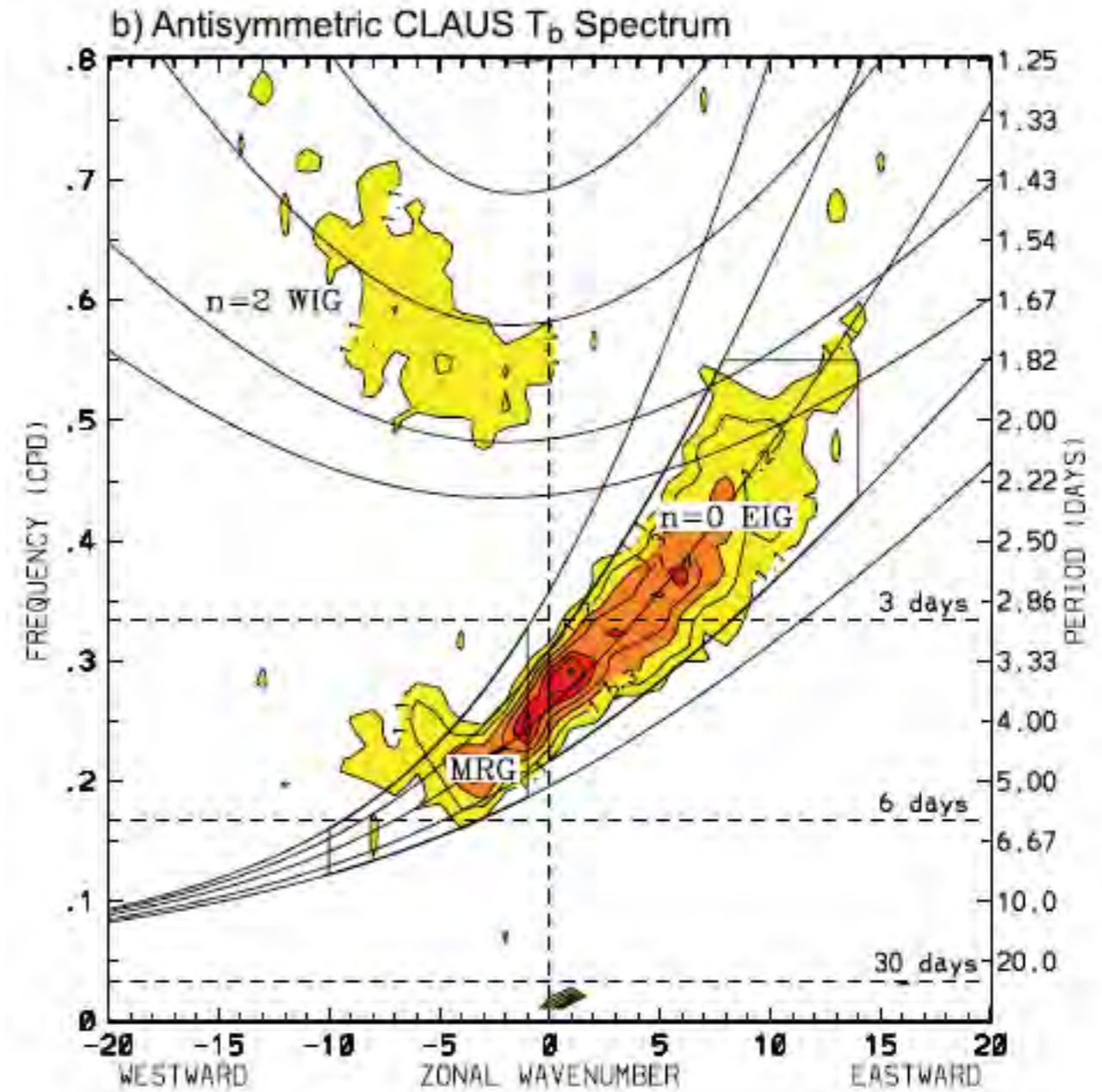
Figure: Caballero and Huber 2010.

# Mechanisms?

To characterize the equatorial variability of the dry dynamical core we use the space-time spectral analysis method of Wheeler and Kiladis 1999.



Kiladis 2009. Symmetric modes



Kiladis 2009. Asymmetric modes



# Equatorial modes

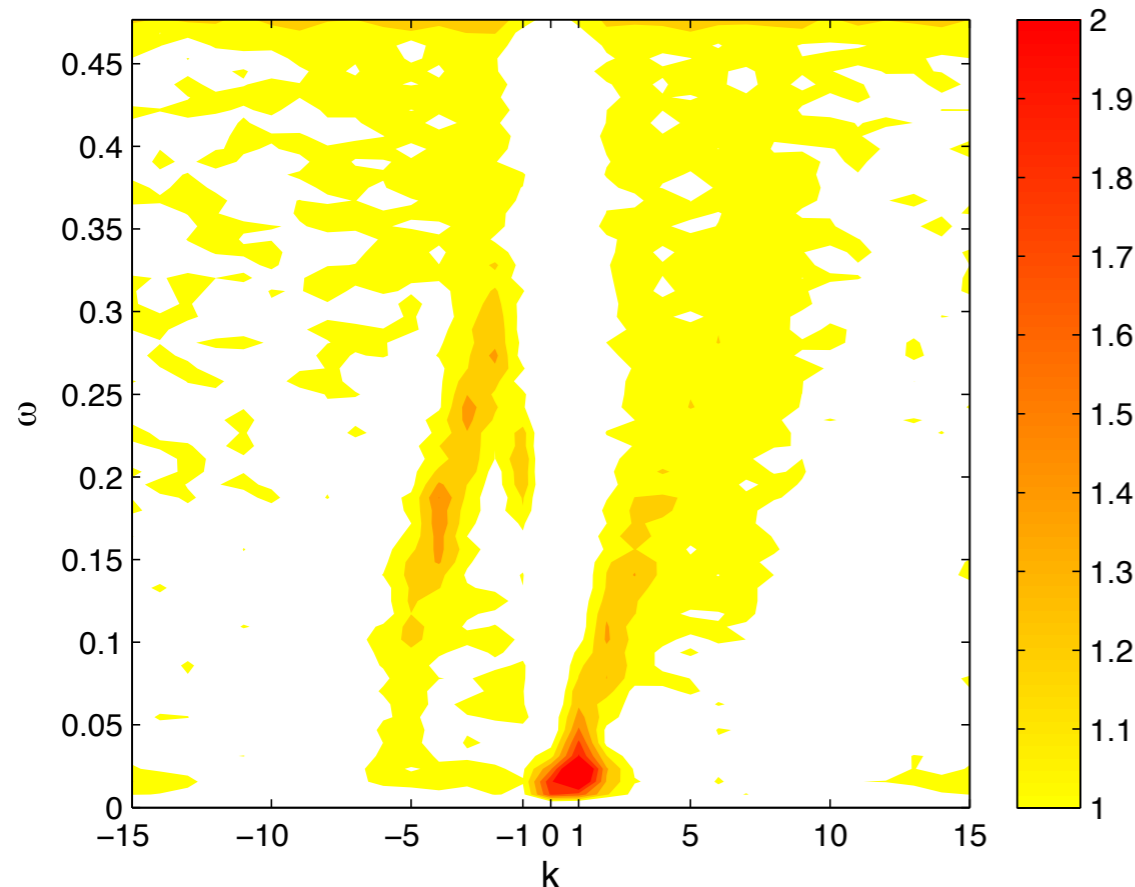
A related way of viewing the equatorial eddy activity is to look at the space-time spectral decomposition of eddy momentum flux convergence (Hayashi 1971).

$$-\frac{\partial}{\partial y} \overline{u'v'}(x, y, t) \rightarrow -\frac{\partial}{\partial y} \overline{u'v'}(k, y, \omega)$$

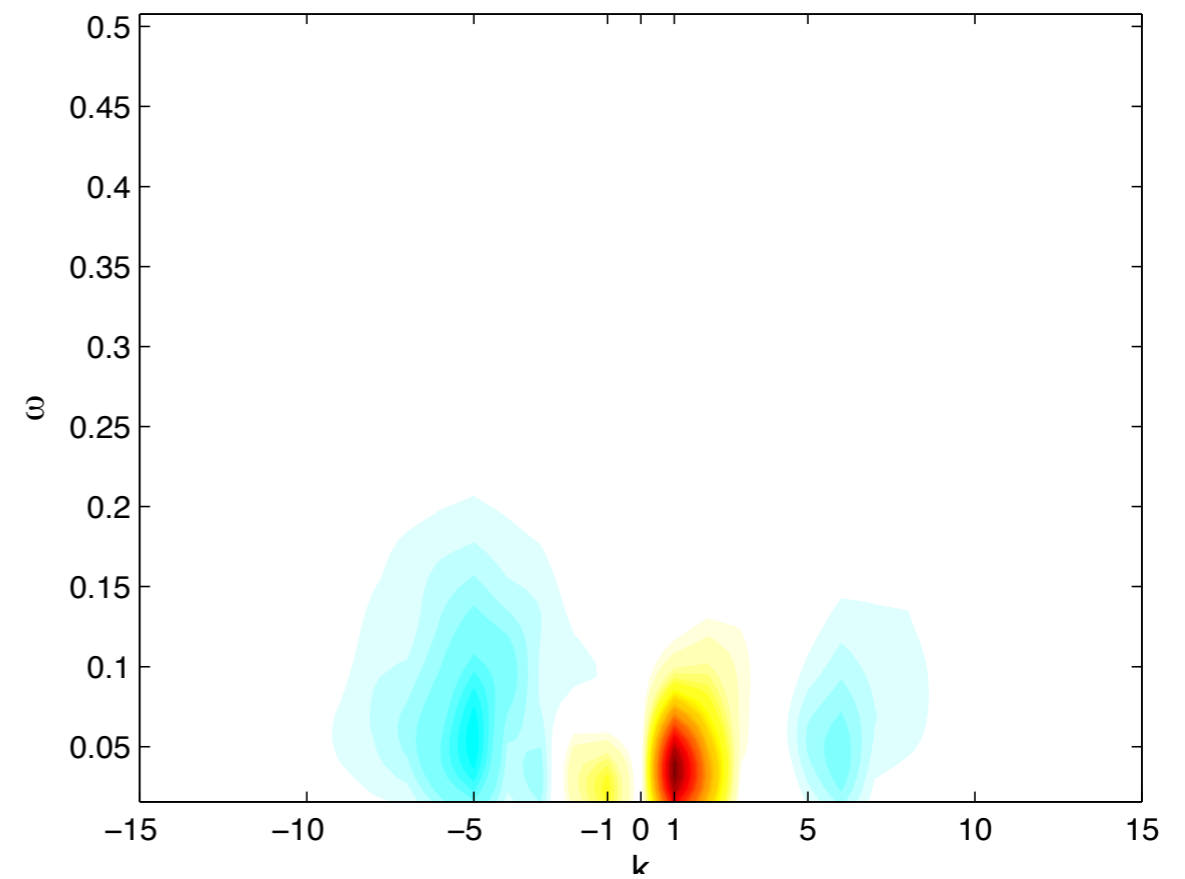
where  $x$  is longitude,  $y$  is latitude,  $t$  is time,  $k$  is wavenumber and  $\omega$  is frequency.

Biggest signal in  $-\frac{\partial}{\partial y} \overline{u'v'}$  is from the MJO, which acts to accelerate the equatorial winds.

Symmetric modes for  $u$  field at 150 mb.



$-\frac{\partial}{\partial y} \overline{u'v'}$  at 150 mb and  $0^\circ$

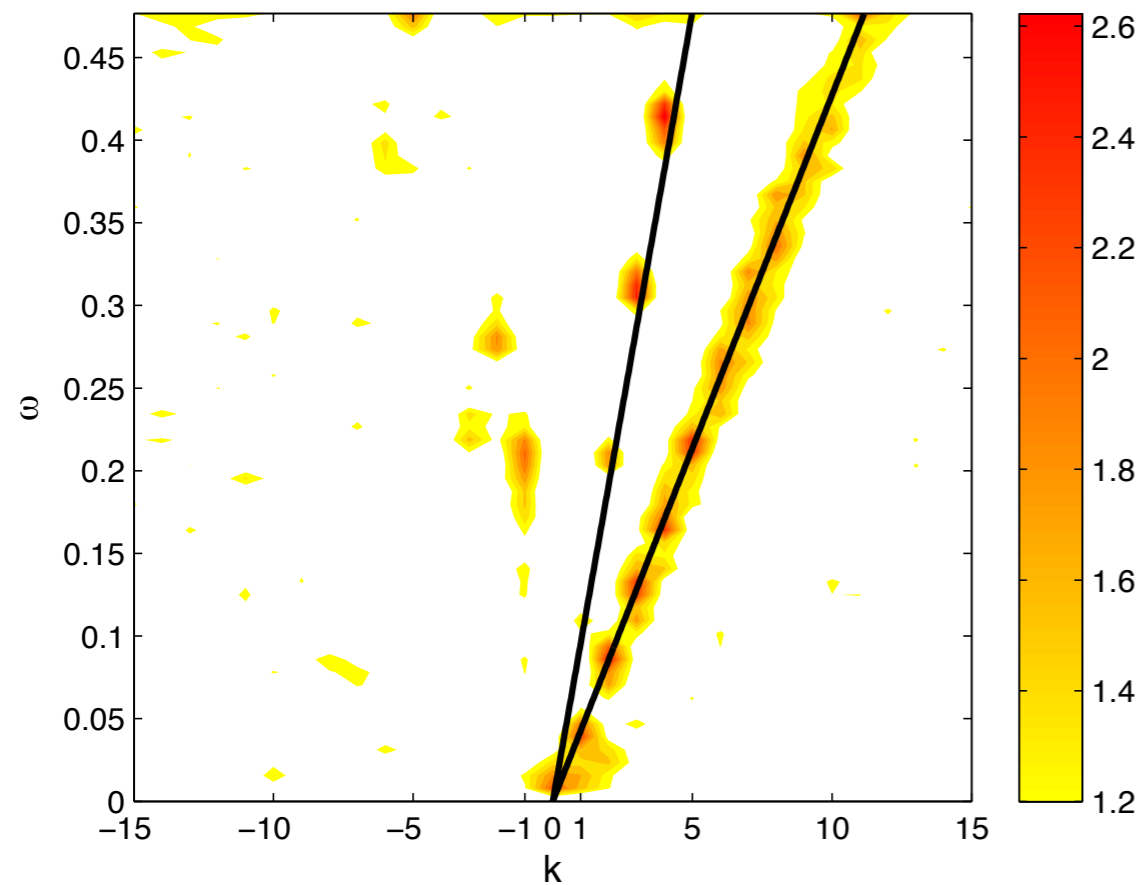


# Equatorial modes

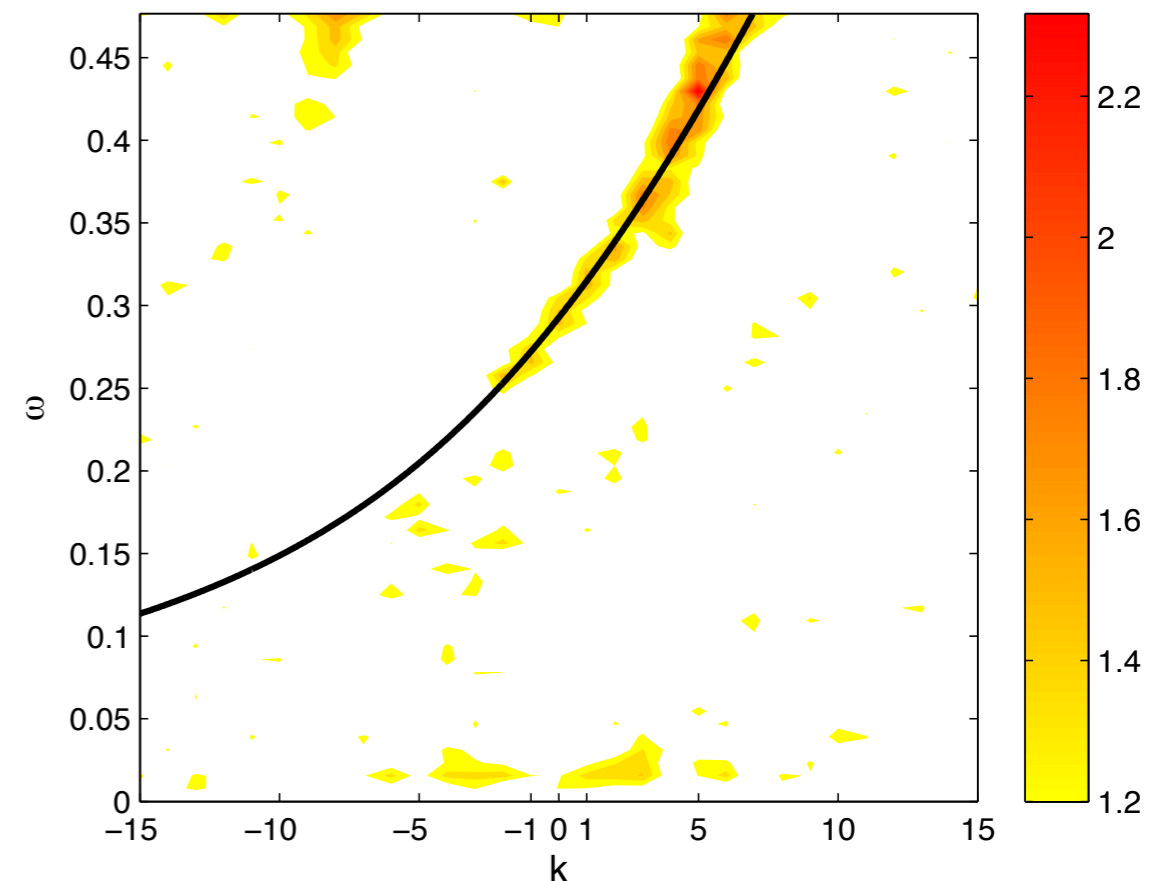
Model results: dry dynamical core for Earth-like parameters

There are strong Kelvin and  $n = 0$  eastward inertio-gravity waves. There is also a hint of Rossby-Haurwitz modes.

Symmetric modes for  $T$  field at 274 mb.  
Kelvin wave dispersion relation for  $H = 40$   
and  $H = 200$



Asymmetric modes for  $T$  field at 274 mb.  
Mixed Rossby-gravity and  $n=0$  eastward  
inertio-gravity dispersion relations for  
 $H = 40$



# *Equatorial modes in a superrotating atmosphere*

$$Ro_T = 0.1$$

Superrotation starts near  $Ro_T = 0.1$ , or where radius  $a = 2.8e6$  m.

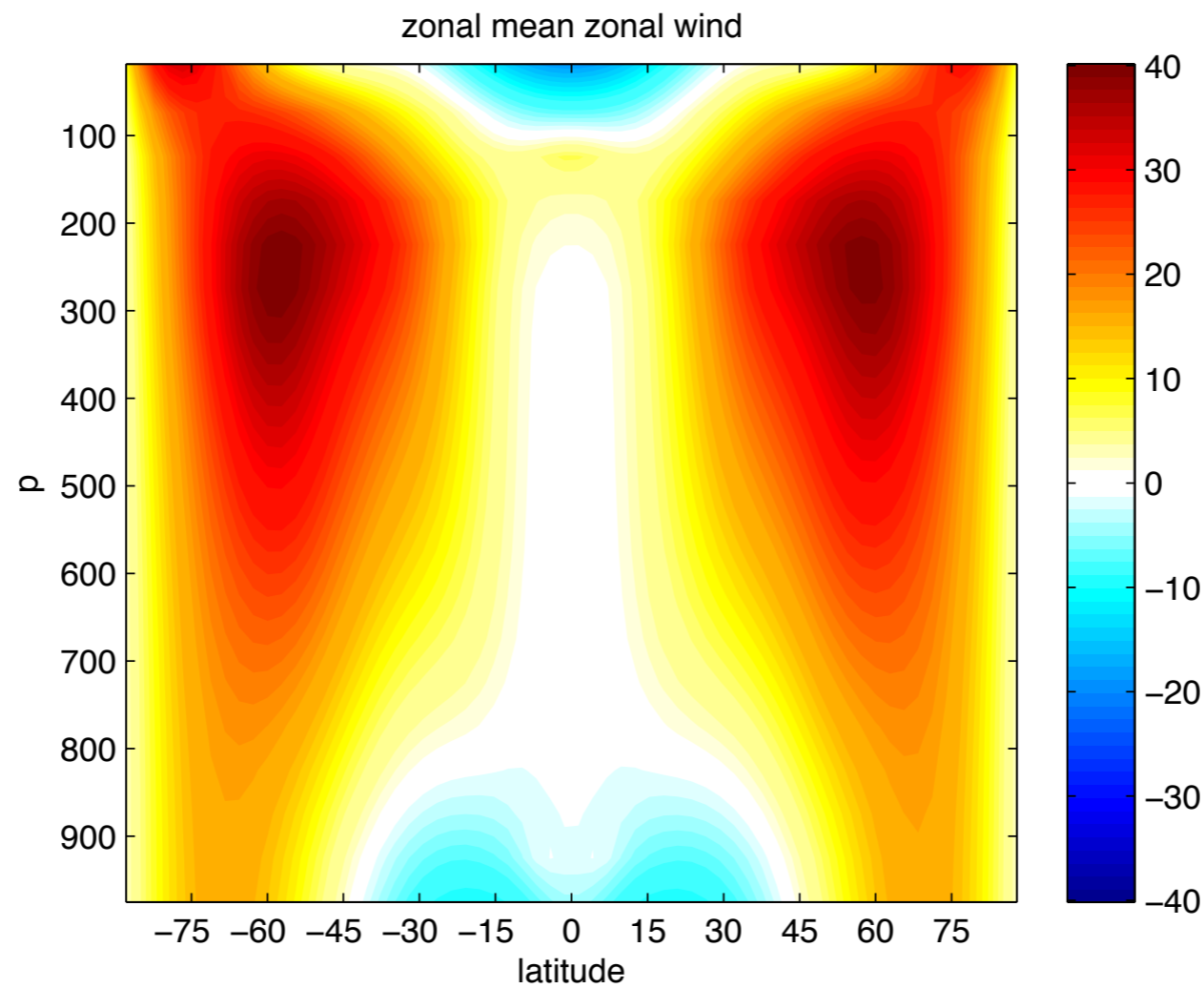
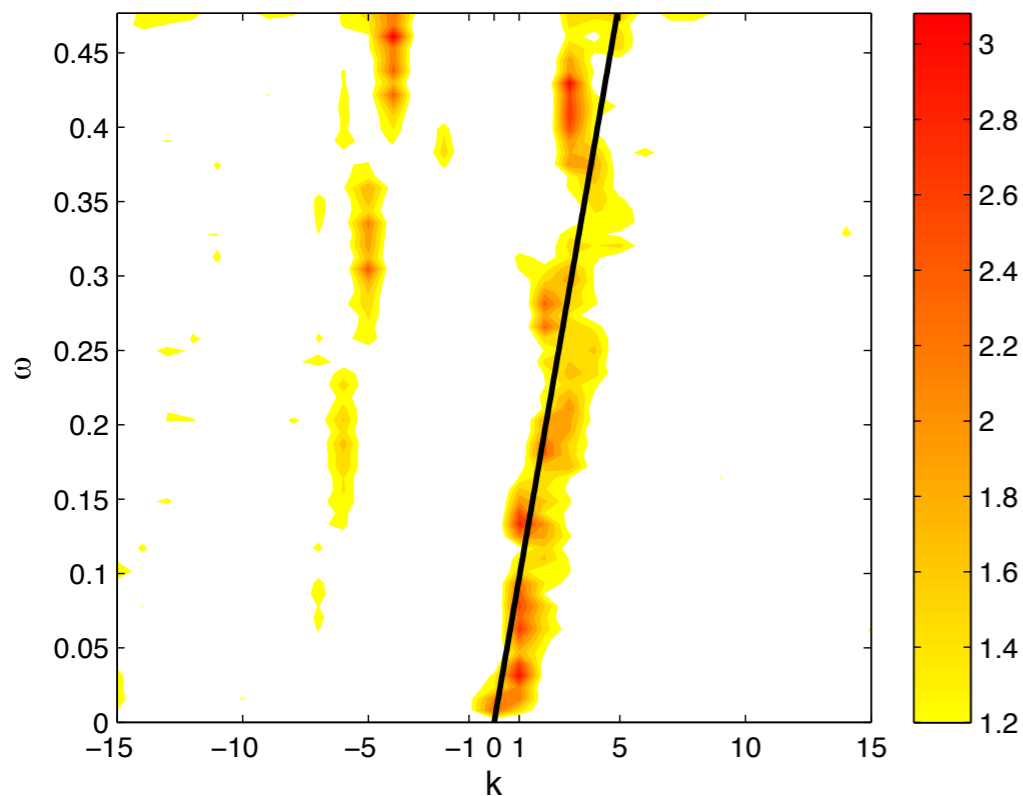


Figure: Zonal mean zonal winds for dynamical core with radius =  $2.8e6$  m.

# Equatorial modes: $Ro_T = 0.1$

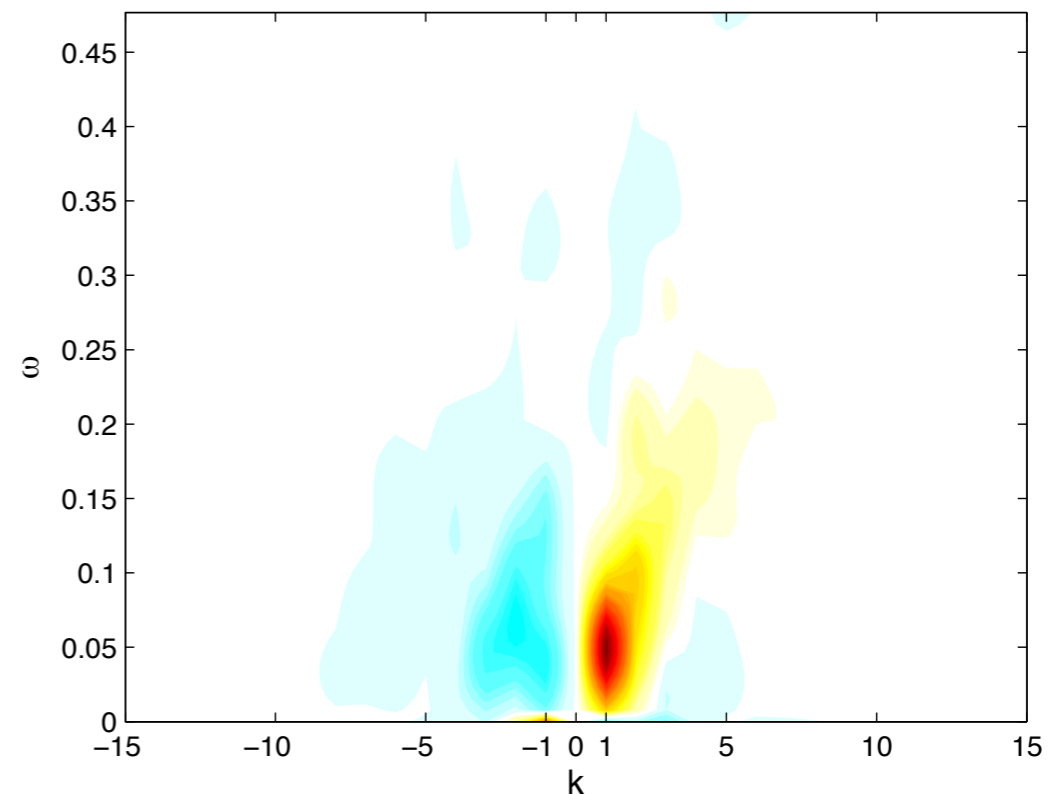
The superrotation is driven by an eastward moving wavenumber-1 mode. There is again deceleration by what is most likely global Rossby waves.

### Temperature variability



Symmetric modes for  $T$  field at 274 mb. Kelvin wave dispersion relation for  $H = 40$

### Momentum convergence

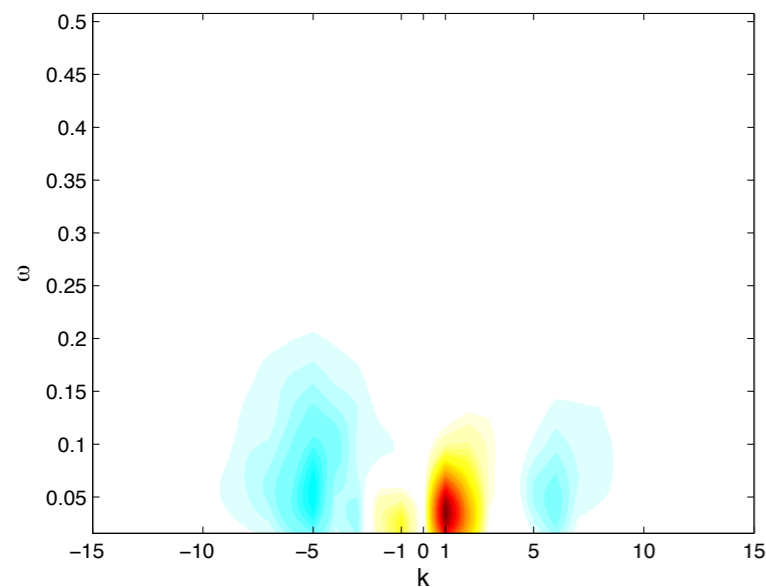


$-\frac{\partial}{\partial y} \overline{u'v'}$  at 274 mb and  $1.3^\circ$

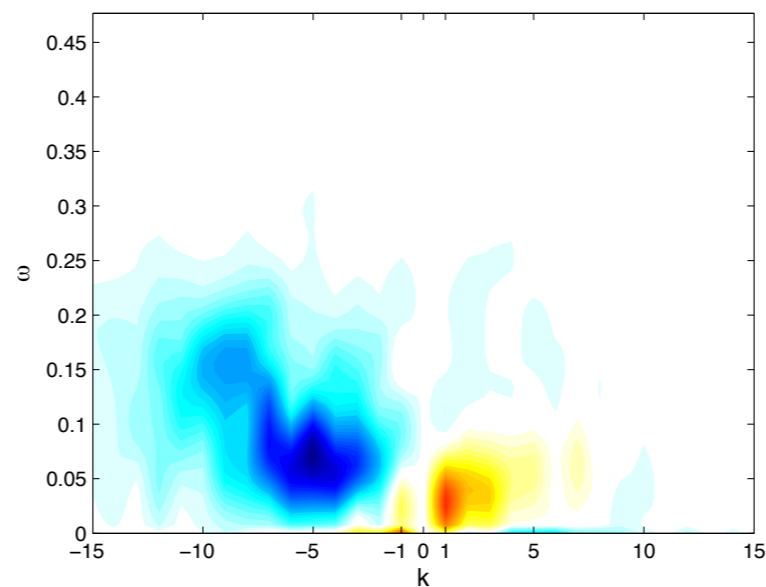
# Equatorial modes

The mode that drives the superrotation falls in a similar location in  $k - \omega$  space to the MJO in reanalysis.

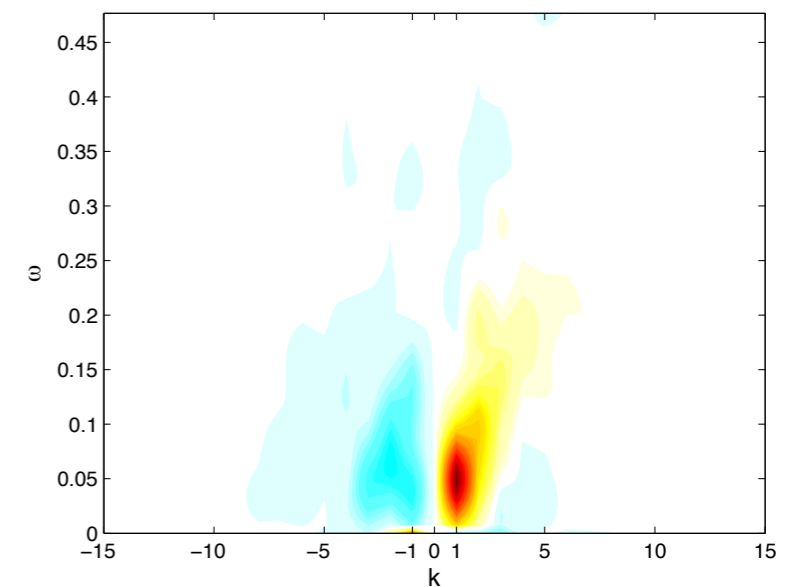
Observations:  $-\frac{\partial}{\partial y} \overline{u'v'}$  at 150 mb and  $0^\circ$



Control:  $-\frac{\partial}{\partial y} \overline{u'v'}$  at 374 mb and  $1.3^\circ$



$Ro_T = 0.1$ :  $-\frac{\partial}{\partial y} \overline{u'v'}$  at 274 mb and  $1.3^\circ$



Appears to be an eastward propagating, MJO-like disturbance that gives rise to superrotation, not Rossby waves generated at the equator.

# Conclusions

- Earth's atmosphere and climate seems the most interesting of all (IMHO). Diverse range of dynamics and physics.
- On terrestrial planets, superrotation is achievable either through:
  - ① Forcing, including convection, at the equator.
  - ② Spontaneous superrotation, especially in 'slowly rotating' regimes.
- Seasonal cycle mitigates against superrotation.
- In hot-house climates, and in slowly rotating regimes, superrotation is associated with an equatorial mode that resembles the Madden Julian Oscillation (MJO).
- Superrotation may be a way of warming the dark side of tidally-locked exoplanets, but the mechanisms may be completely different.