Radiative-Convective Modeling of Planetary Climate

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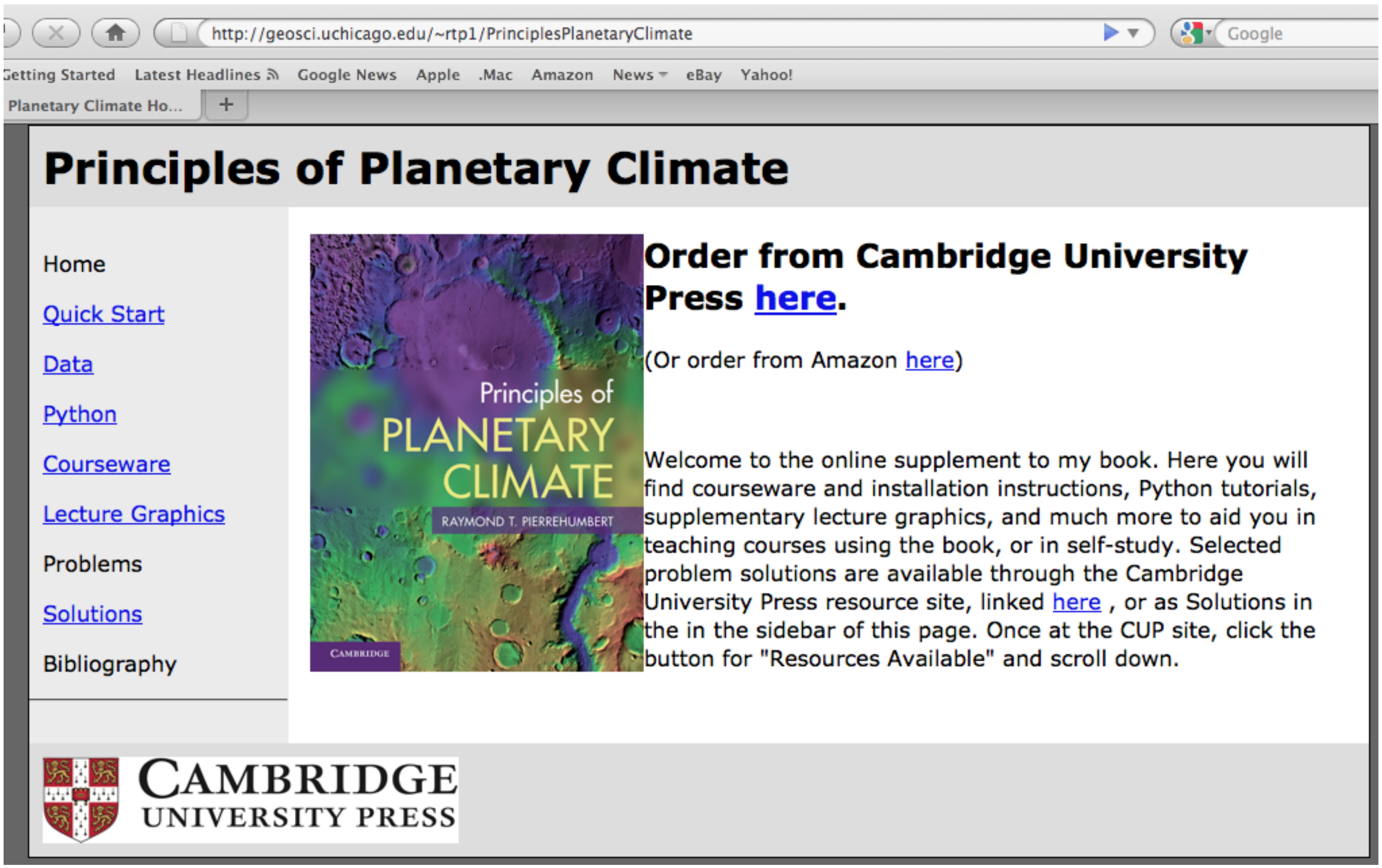
Outline of the Lecture

- Basic formulation and solution methods
- Some interesting applications
- Beyond 1D: The need for dynamics

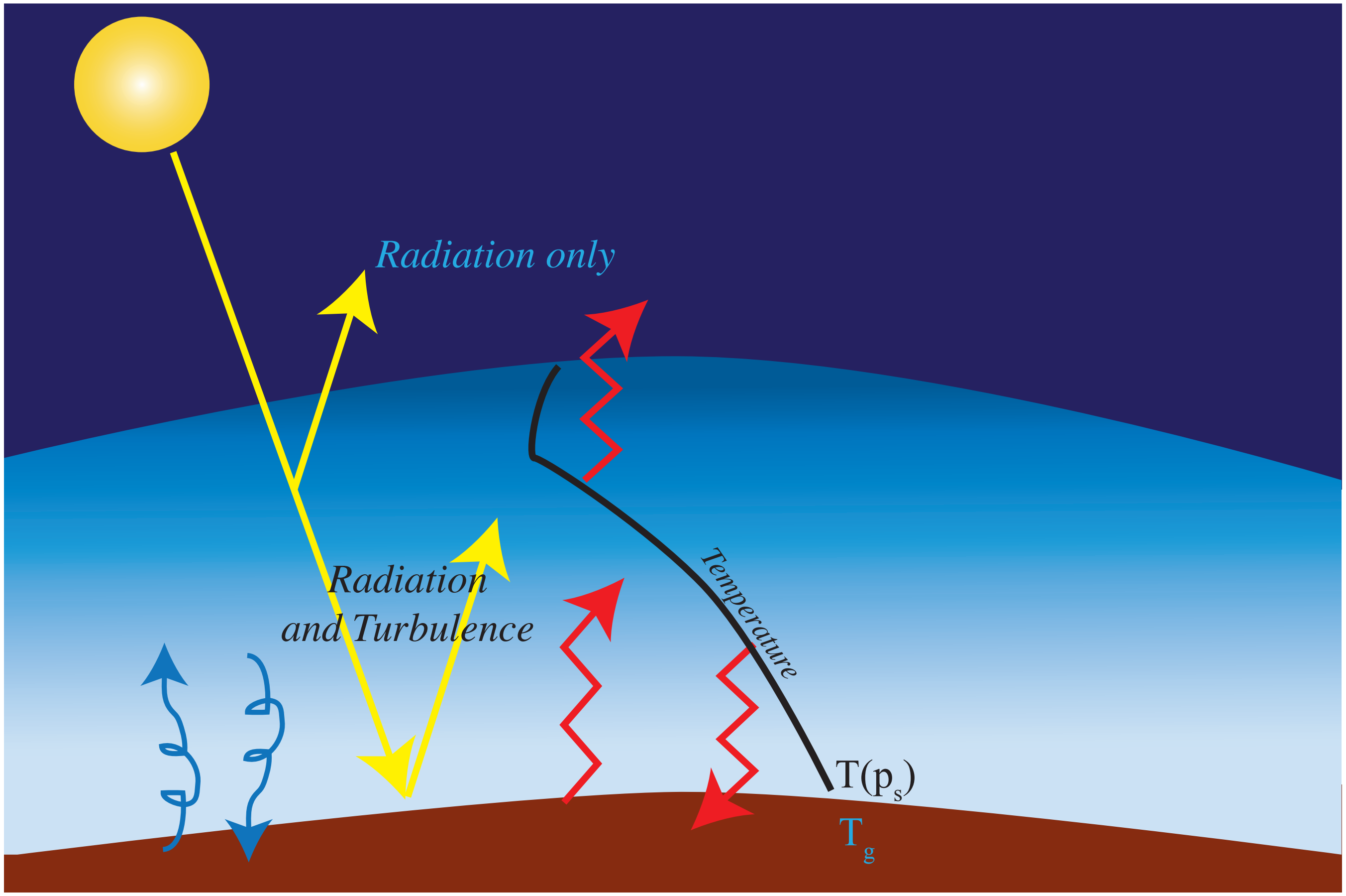
Basic formulation: What is a radiative convective model?

- Represent entire atmosphere by a single vertical column $T(p,t)$, etc.
- Column generally meant to represent global mean climate
- Only vertical energy transport is modeled
  - Radiative transport
  - Turbulent transport due to convection
  - Convection modeled as a 1D mixing process

For further reading ...



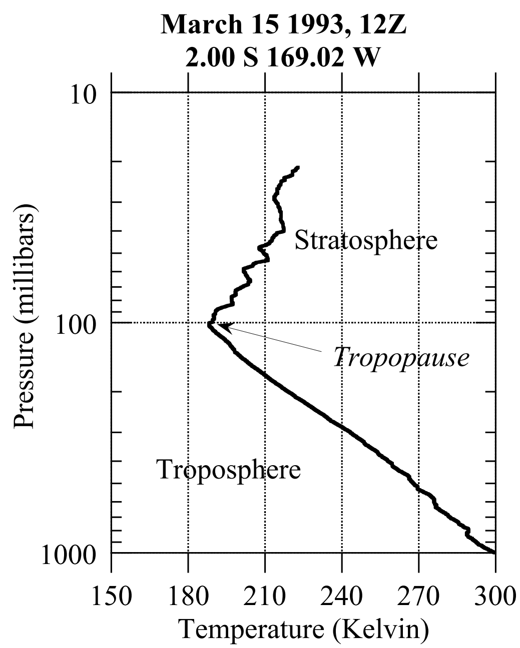
Radiating temperature and the greenhouse effect

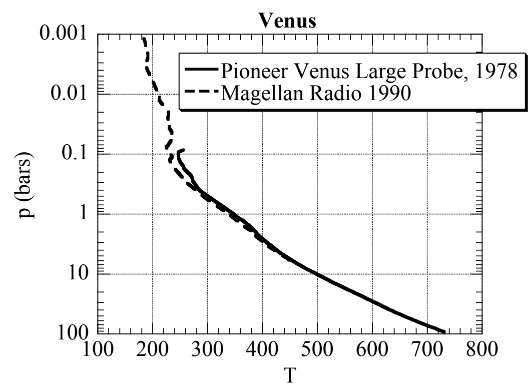


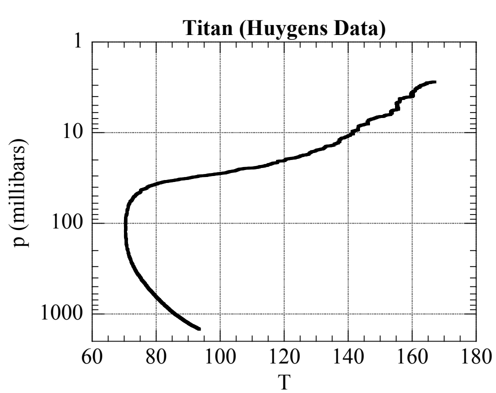
For many purposes, can assume $T_g \approx T(p_s)$

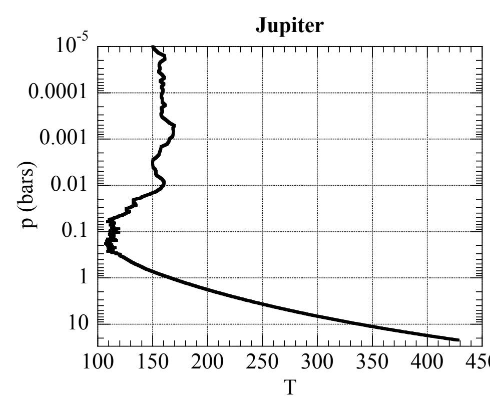
For review, see: Pierrehumbert 2010: Infrared Radiation and Planetary Temperature, *Physics Today* **64**

A few observed vertical structures









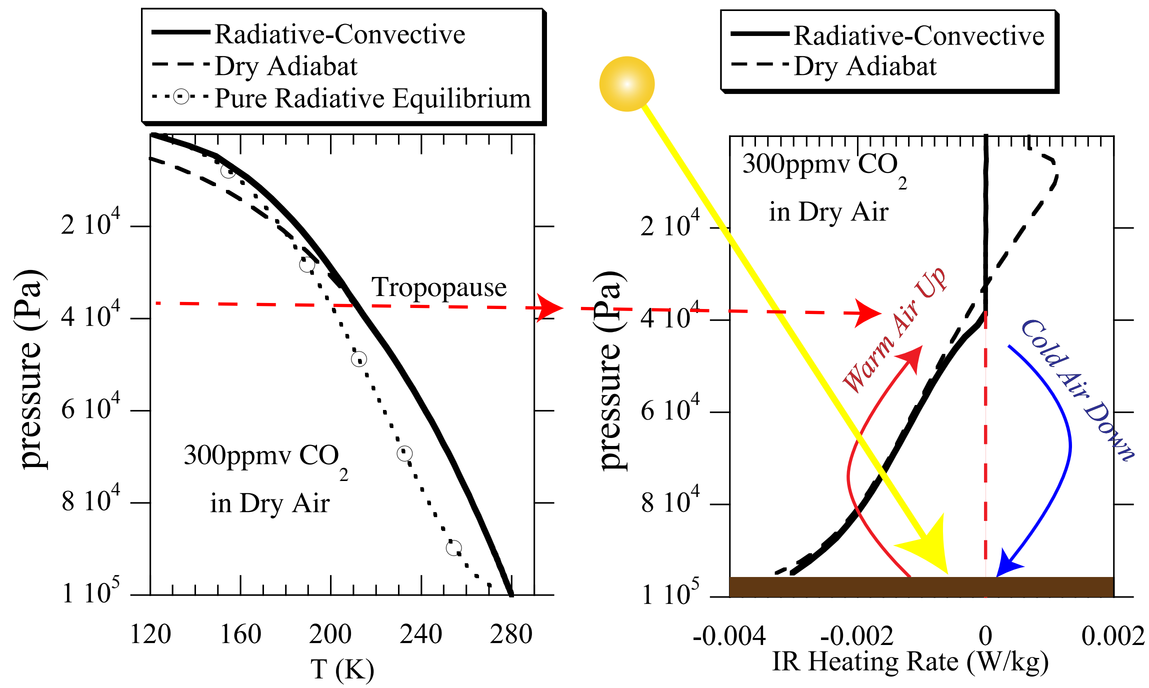
The IR Radiative Transfer model

- Input $T(p)$, composition (e.g. p_{CO2},$q(p)$)
- $\longrightarrow$ fluxes $I_+(p)$, $I_-(p)$
- Heating rate $H = g^{-1} d(I_+ - I_-)/dp$

Pure radiative equilibrium

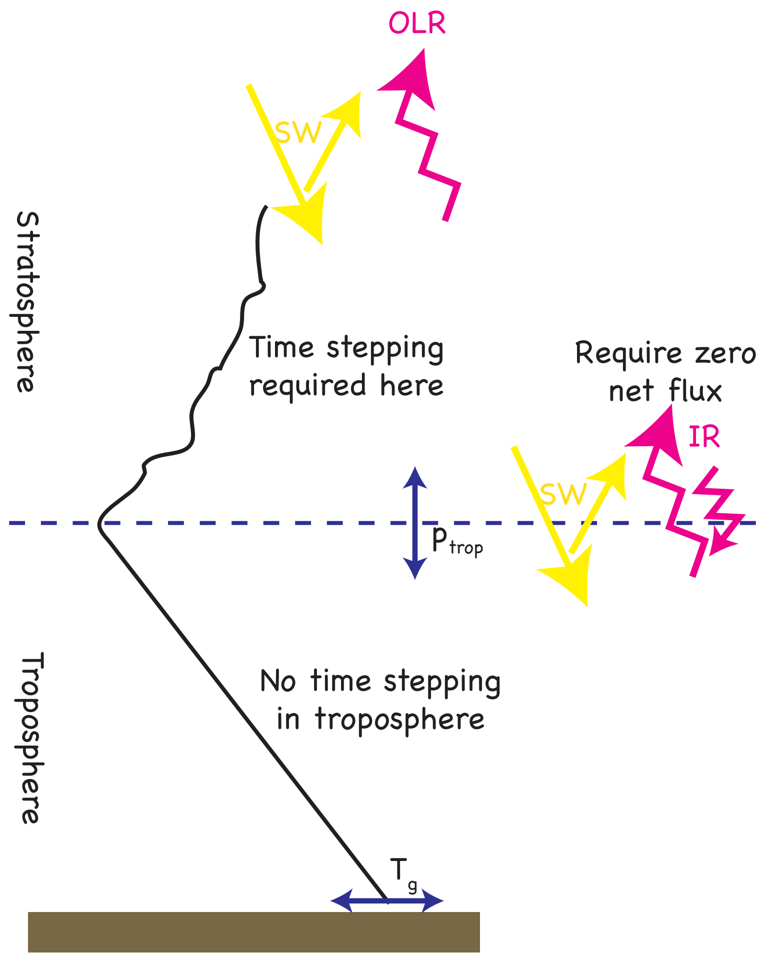
- $H + H_{sw} = 0$ at each p
- Equivalently $I_+ - I_- + F_\circledast = \text{const.}$
- Typically ignore scattering for IR, but incorporate it for $F_\circledast$
- Wavelength of incoming stellar radiation $\ll$ Wavelength of outgoing IR
- ... but this separation can break down for roasters.

Energy balance for atmosphere transparent to incoming stellar radiation



$(\int_{p_t}^{p_s} H dp/g) + S_{abs} = 0$; Determines tropopause height

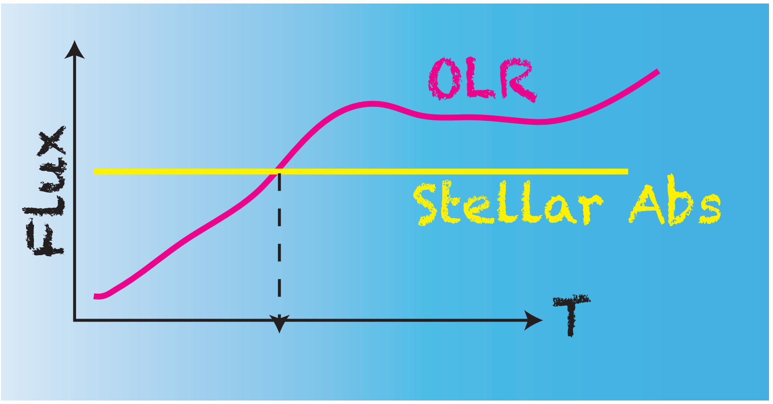
How to get $OLR(T_g)$



- Without atmospheric shortwave absorption, stratospheric temperature depends on insolation only via T_g.

- For typical atmospheres stratosphere is optically thin in IR. Can then use isothermal stratosphere or ”all troposphere model” and dispense with time-stepping entirely.

Once you have $OLR(T_g)$ ...



Plot it, and you're done!

When is a 1D model sufficient?

$$< OLR(T_g, q) > \approx OLR(< T_g >, < q >)$$

$$+\frac{1}{2}(\partial_{TT} OLR) < T'^2 > +\frac{1}{2}(\partial_{qq} OLR) < q'^2 > +(\partial_{Tq} OLR) < T'q' >$$

Forms of convective adjustment

- If you are time-stepping only to find an equilibrium, then the convective adjustment stage need not conserve energy
- If you are trying to represent the actual time evolution (e.g. the diurnal or seasonal cycle) the convective adjustment stage needs to conserve energy

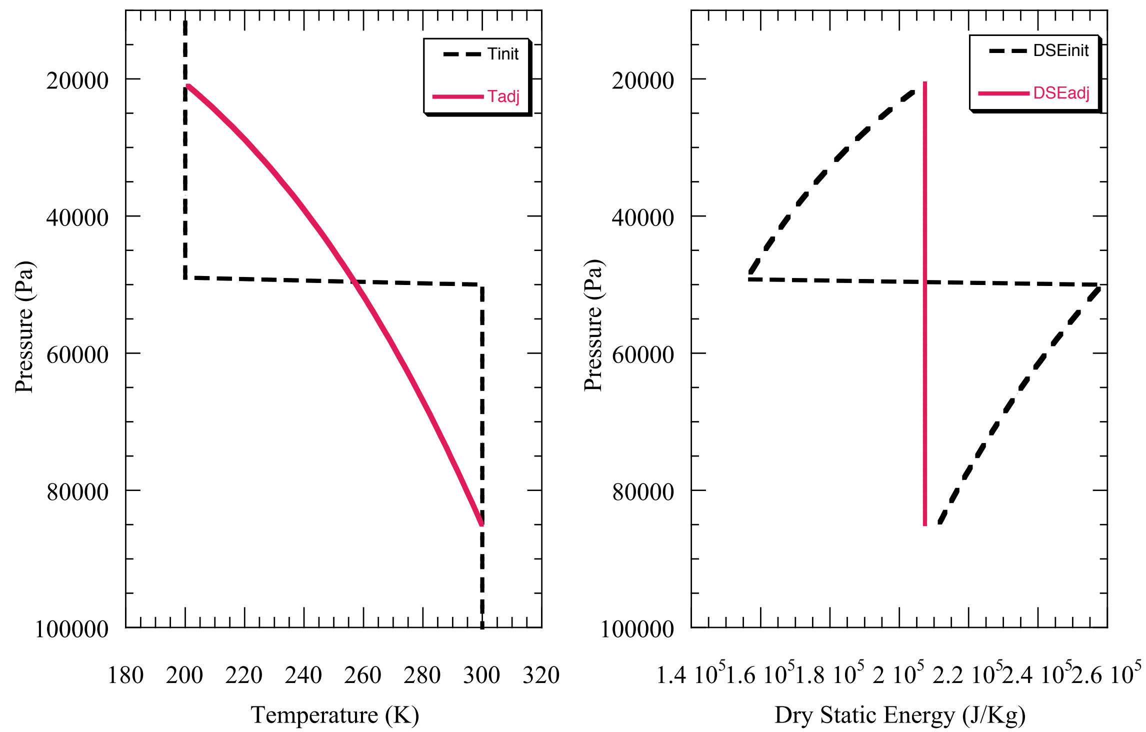
What is conserved during convective adjustment?

- Suppose the adjustment takes place in a layer from p_1 to p_2
- The final state is an adiabat. *Which* adiabat? (e.g. dry adiabat is a one-parameter family defined by $T(p) = T(p_1)(p/p_1)^{R/c_p}$)
- First Law: $T^{-1}\delta Q = ds = dc_p \ln \theta$
- But during adjustment, $\delta Q \neq 0$ at each p. Only have $\int_{p_1}^{p_2} \delta Q dp = 0$
- Therefore $\int_{p_1}^{p_2} T^{-1}\delta Q dp \neq 0$; Entropy does not "mix"

The answer: Dry or Moist Static Energy

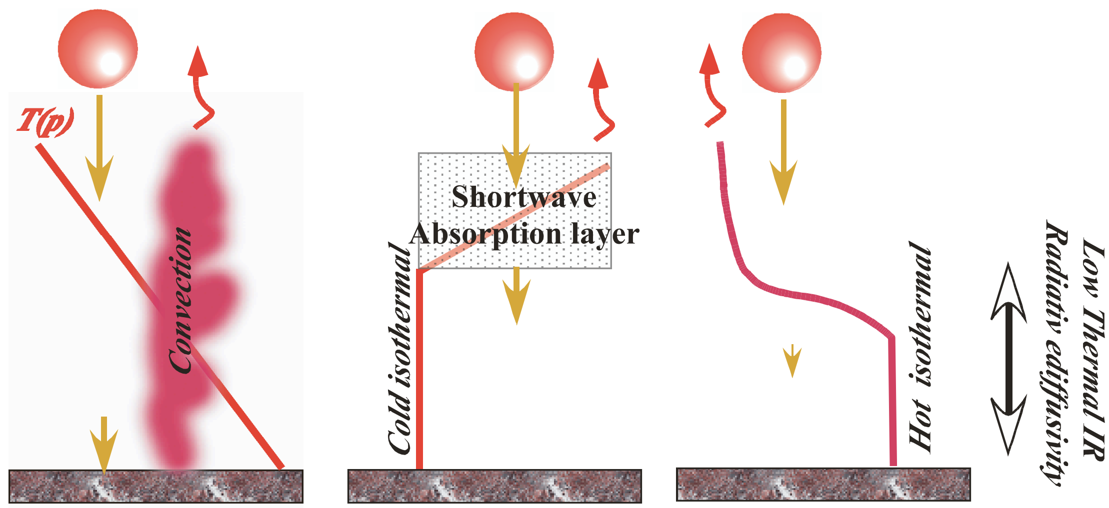
- Define Z(p) from hydrostatic relation
- First law: $\delta Q = d(c_p T + gZ)$ (in dry case)
- Therefore DSE $\equiv \int_{p_1}^{p_2} (c_p T + gZ) dp/g$ conserved during adjustment
- If q is the mass concentration of the condensible, then in dilute limit ($q \ll 1$), MSE density is $c_p T + gZ + Lq$
- Things get interesting (and somewhat unexplored) in the non-dilute limit, where you need to track the energy carried by the condensate.

Example: DSE-conserving mixing of a step discontinuity



Unique adjusted solution for any p_1 and p_2. Adjust p_1 and p_2 to make the temperature continuous at endpoints. (An *assumption* about how convection works!)

Deep atmospheres, optically thick in stellar spectrum

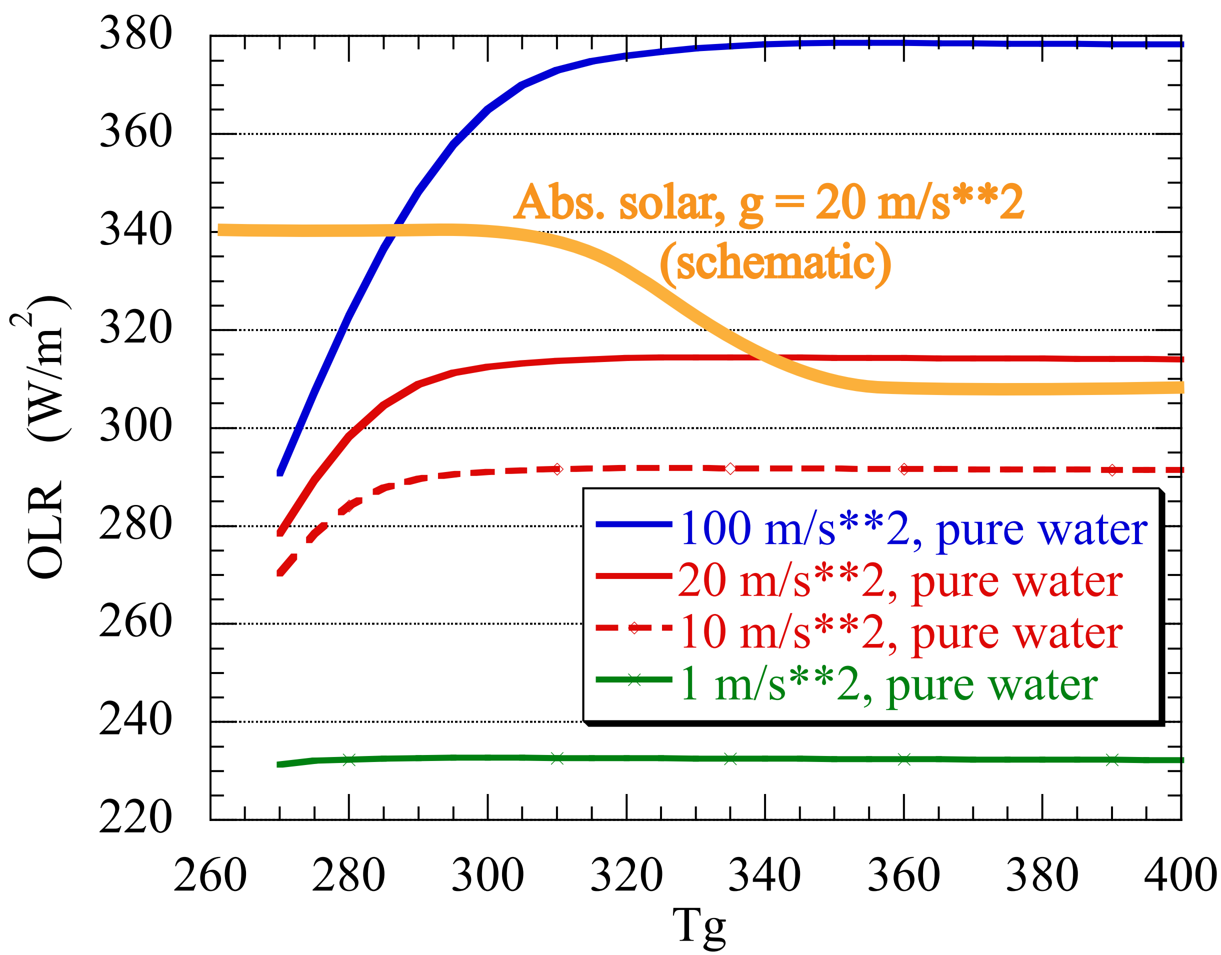


No net flux in deep layer → isothermal

Applications: The conventional habitable zone

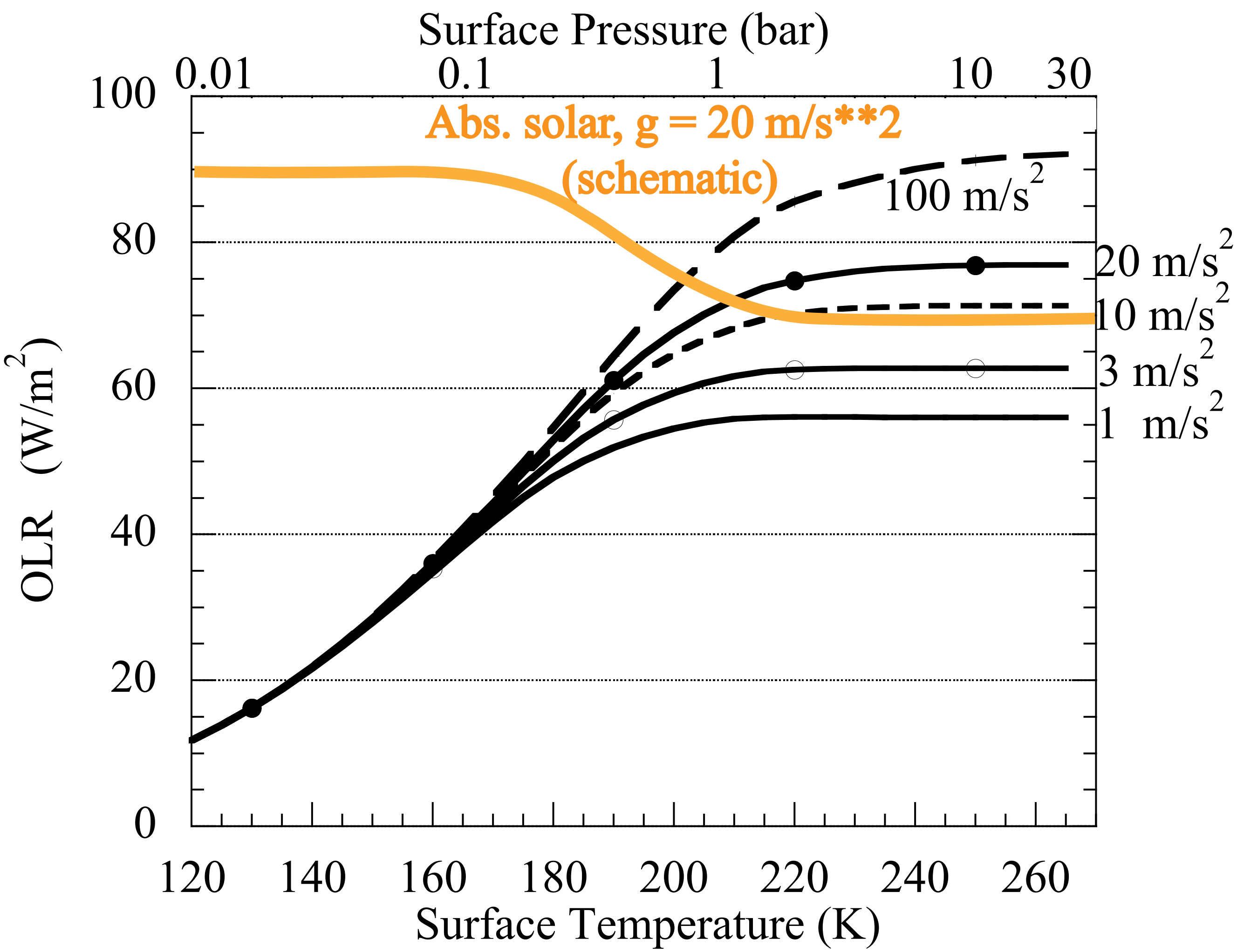
- Inner edge: Water vapor runaway (”wet” or ”dry” version).
  (Runaway = Uninhabitable)
- Outer edge: CO_2 runaway.
  (Runaway = Habitable)

Water Vapor runaway and inner edge (pure WV atmosphere)



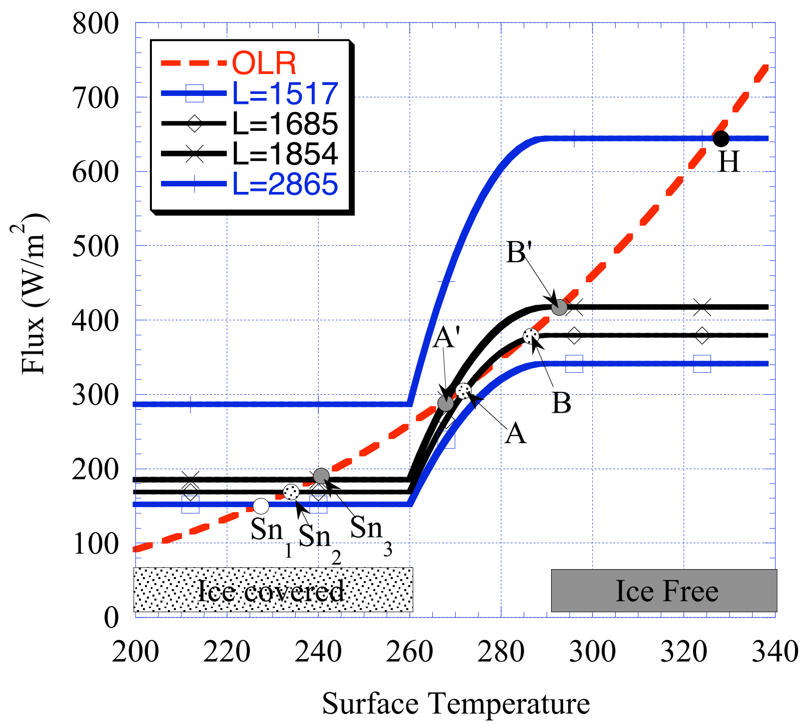
More on this from Colin!

CO_2 runaway and outer edge edge (pure CO_2 atmosphere)



Ch. 4, *Principles ...* plus in-prep radiation model intercomparison by Pierrehumbert, Abbot and Halevy

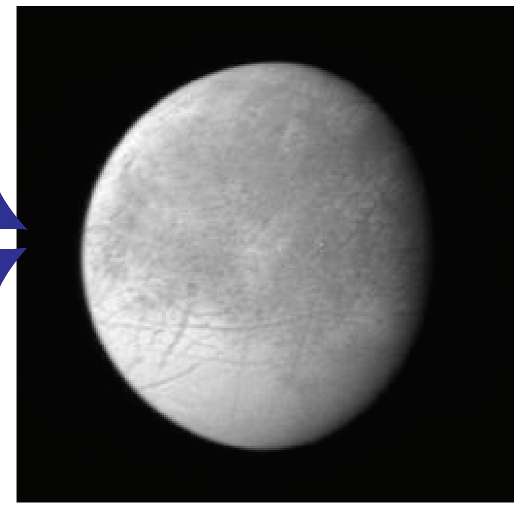
Outer edge: Ice-albedo bifurcation

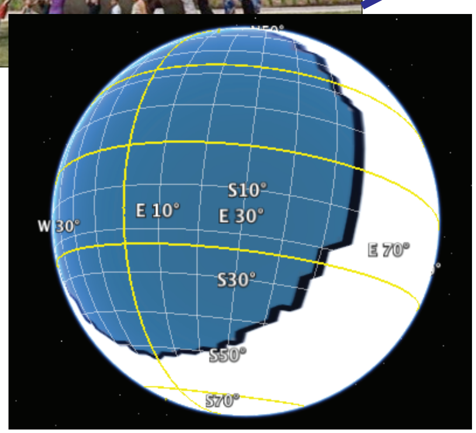


$$\frac{1}{4}(1-\alpha(T))L_{\circledast} = OLR(T, \mathrm{CO_2})$$

Snowball Earths

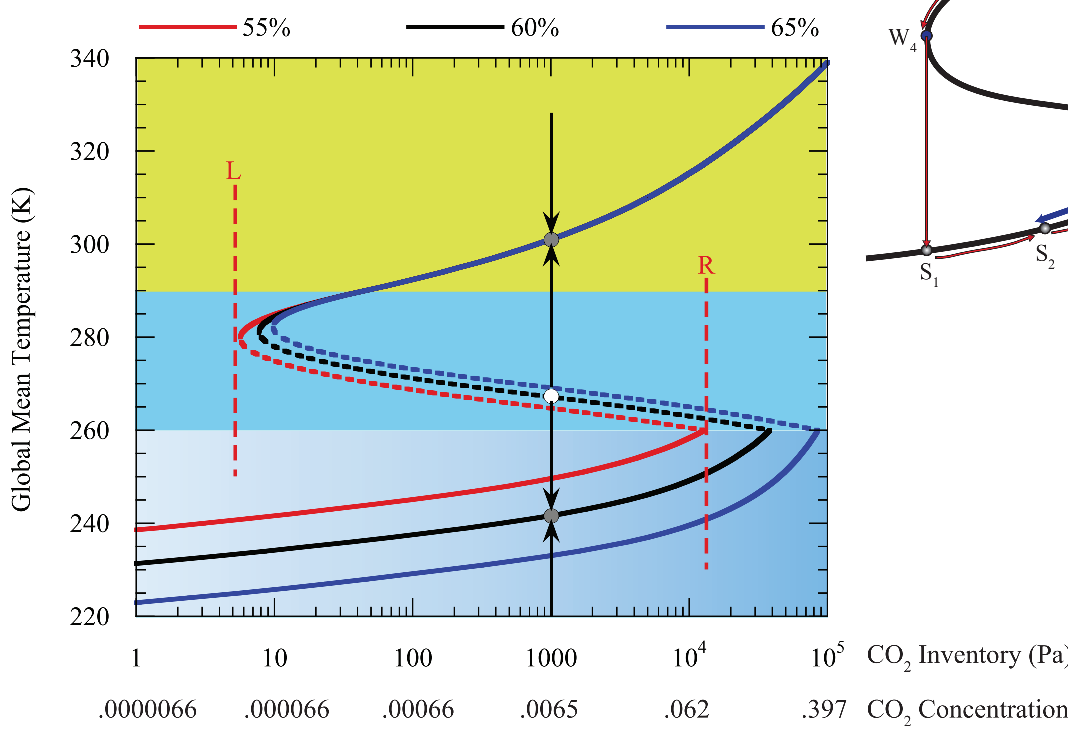
Pierrehumbert , Abbot, Voigt & Koll,
Ann. Rev. Earth and Plan. Sci. 2011

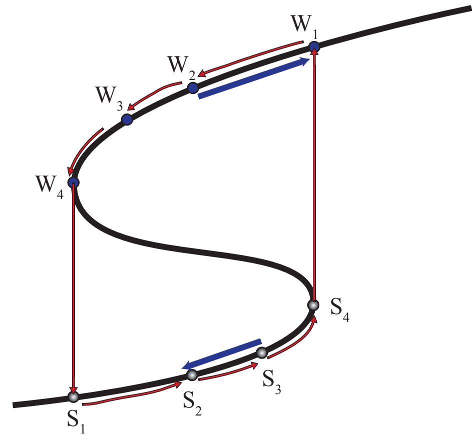




Pierrehumbert , ***Ap. J. L. 2011***

Zero-D Snowball Bifurcation



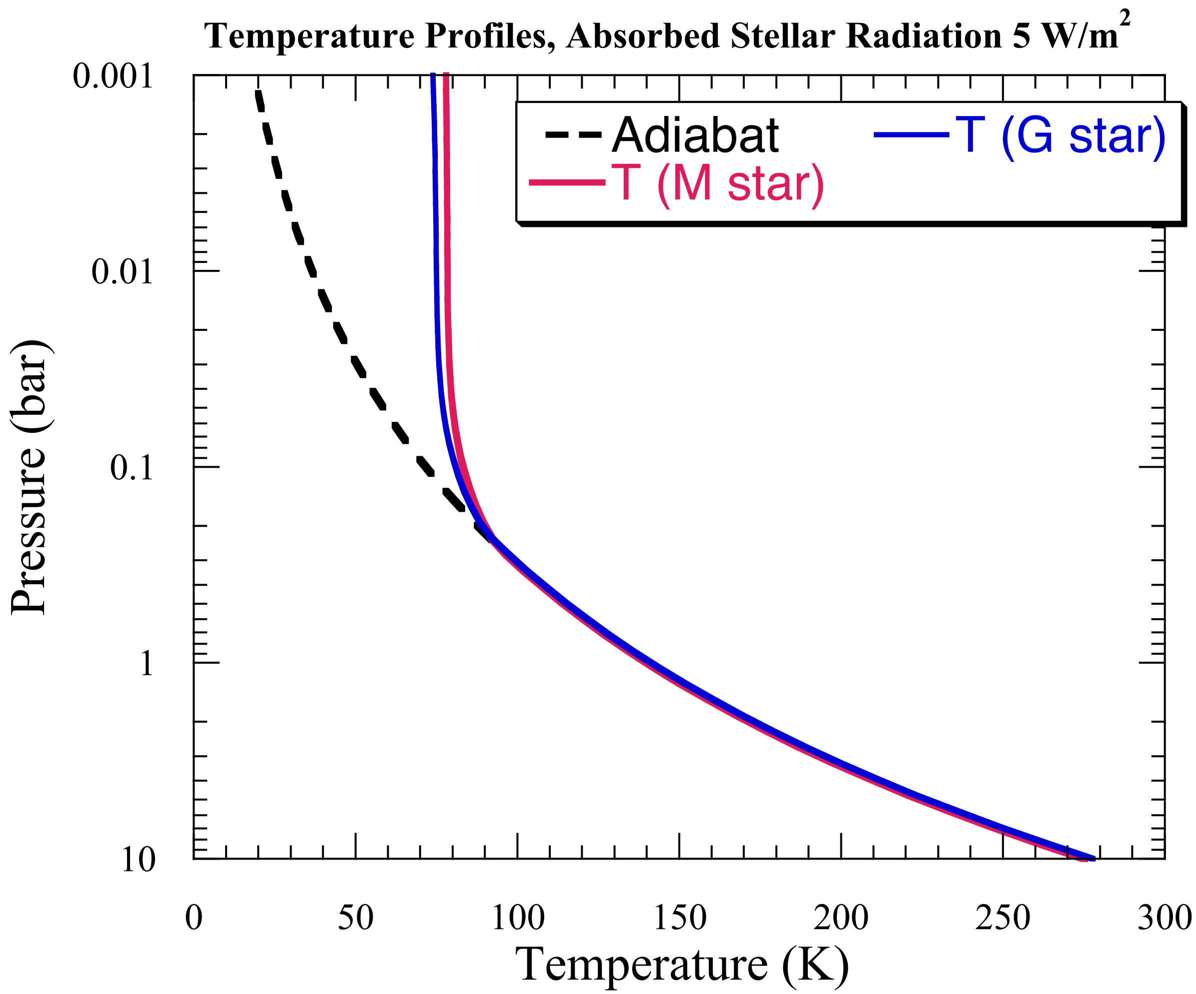


Pierrehumbert, Abbot, Voigt & Koll, *AREPS* 2011

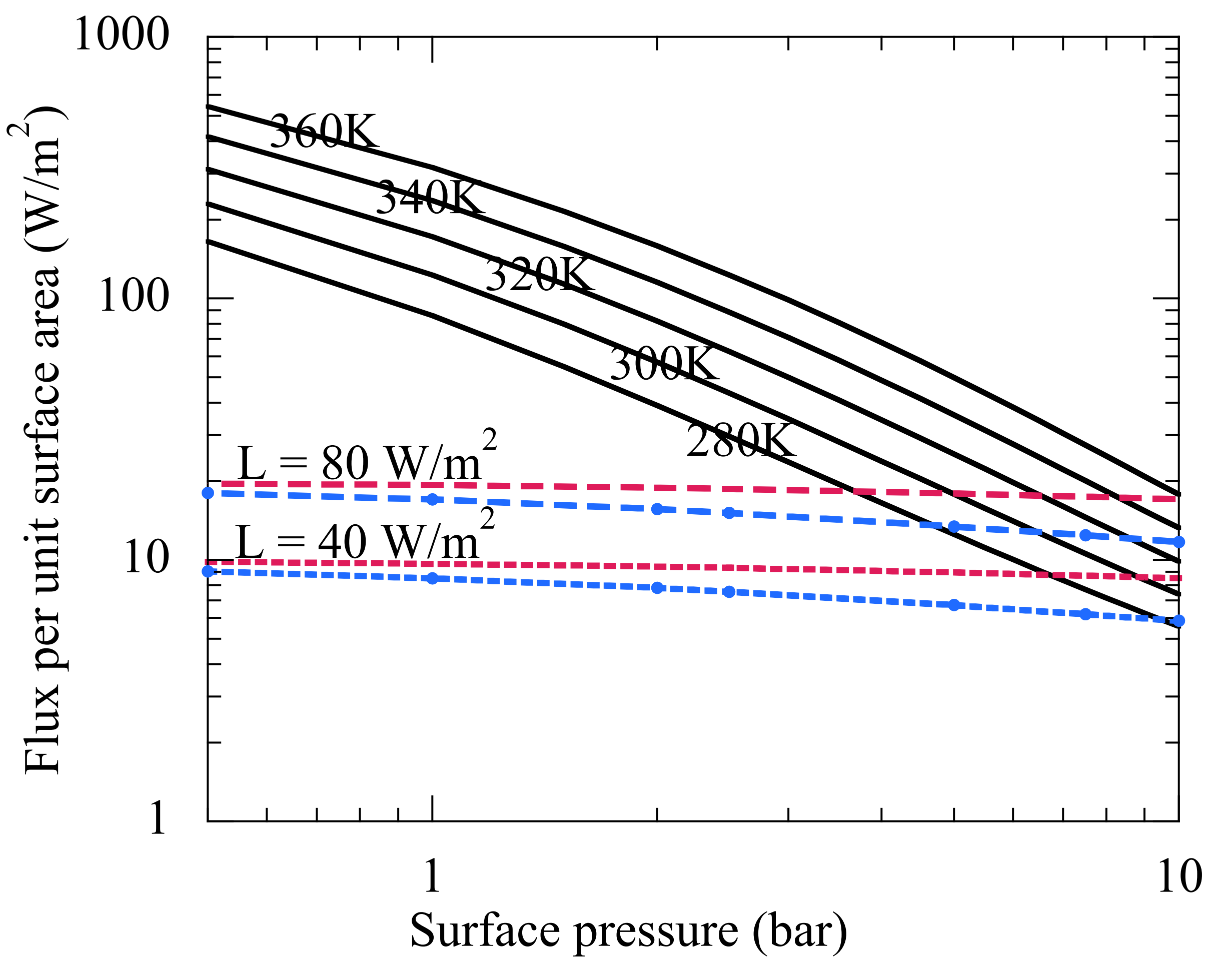
Applications: H_2 worlds

- Conventional outer limit defined by CO_2 runaway. Yields Early Mars equivalent orbit
- To make planets in more distant orbits habitable you need a less condensible greenhouse gas
- So how about H_2?
- In a distant orbit, a Super-Earth can hold an H_2 atmosphere.
- Gravitational lensing has detected Super-Earths in distant orbits.
- Pierrehumbert and Gaidos, *ApJL* 2011
- Also relevant to Steppenwolf planets

Pure H_2 atmosphere



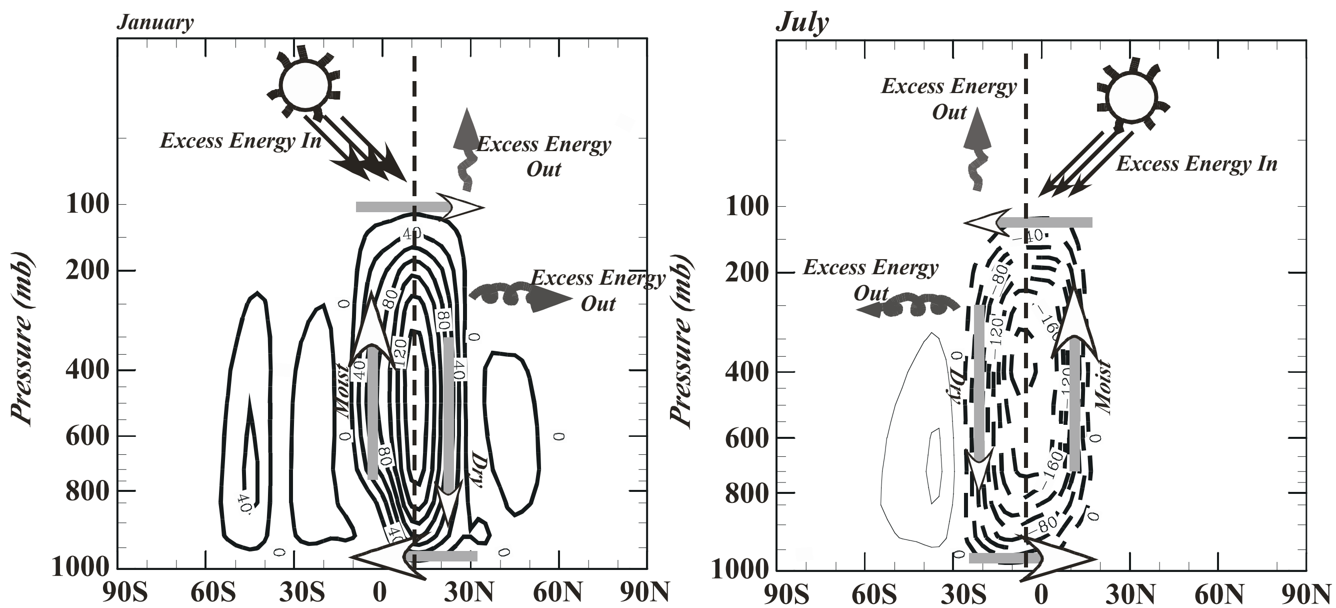
Top-of-Atmosphere Energy Balance



Beyond 1D: The role of large scale dynamics

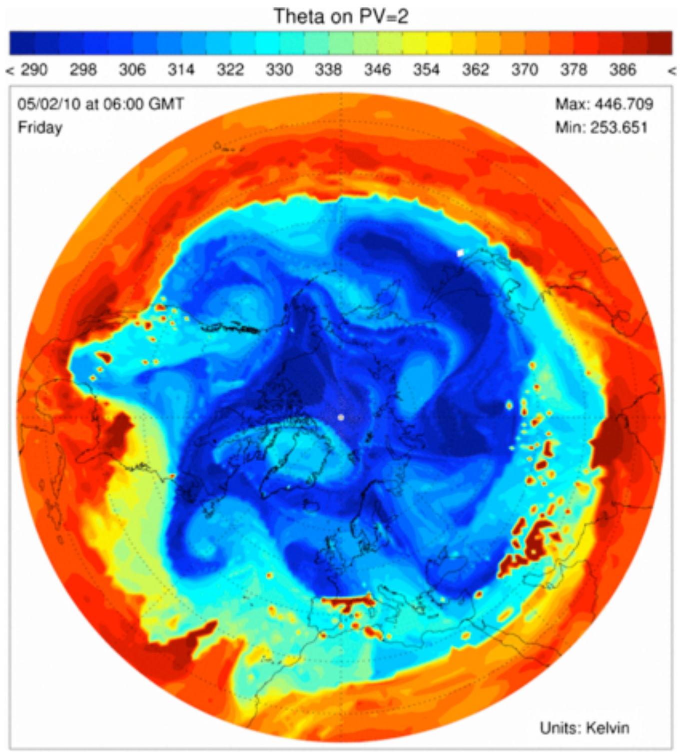
- Horizontal heat transport
- Lapse rate
- Subsaturation (Relative Humidity)
- Clouds

Intro to General Circulation: The Hadley Cell



Gets more global for slow rotation, small planet (Venus,Titan); more equatorially confined for rapid rotation, large planet (Jupiter, Saturn). Earth is "Mr. In-Between."

Intro to General Circulation: Extratropical synoptic eddies

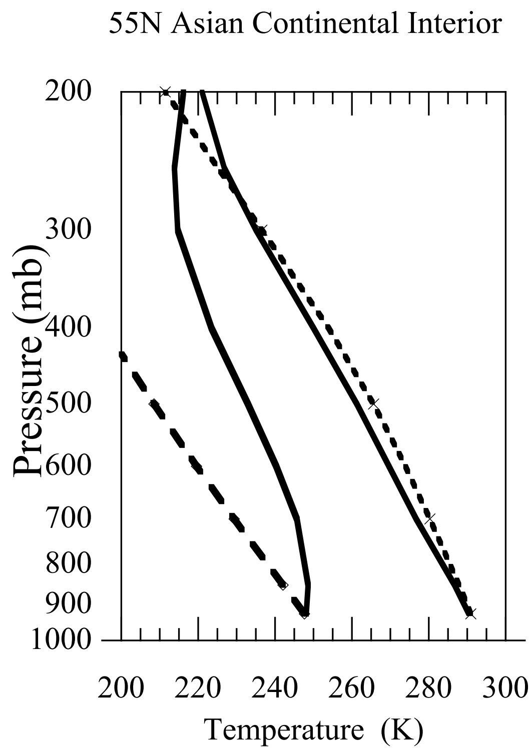


Transport hot air poleward/upward, cold air equatorward/downward.

Beyond 1D: Lapse Rate

- Hadley cell sets the entire tropics to the moist adiabat, even though convection is active in only a small proportion of the tropics
- In midlatitudes, there may be little convection, and lapse rate is determined by large scale transports of heat by transient baroclinic eddies. Lapse rate can have large geographic and seasonal variation.

Example: Siberian lapse rate

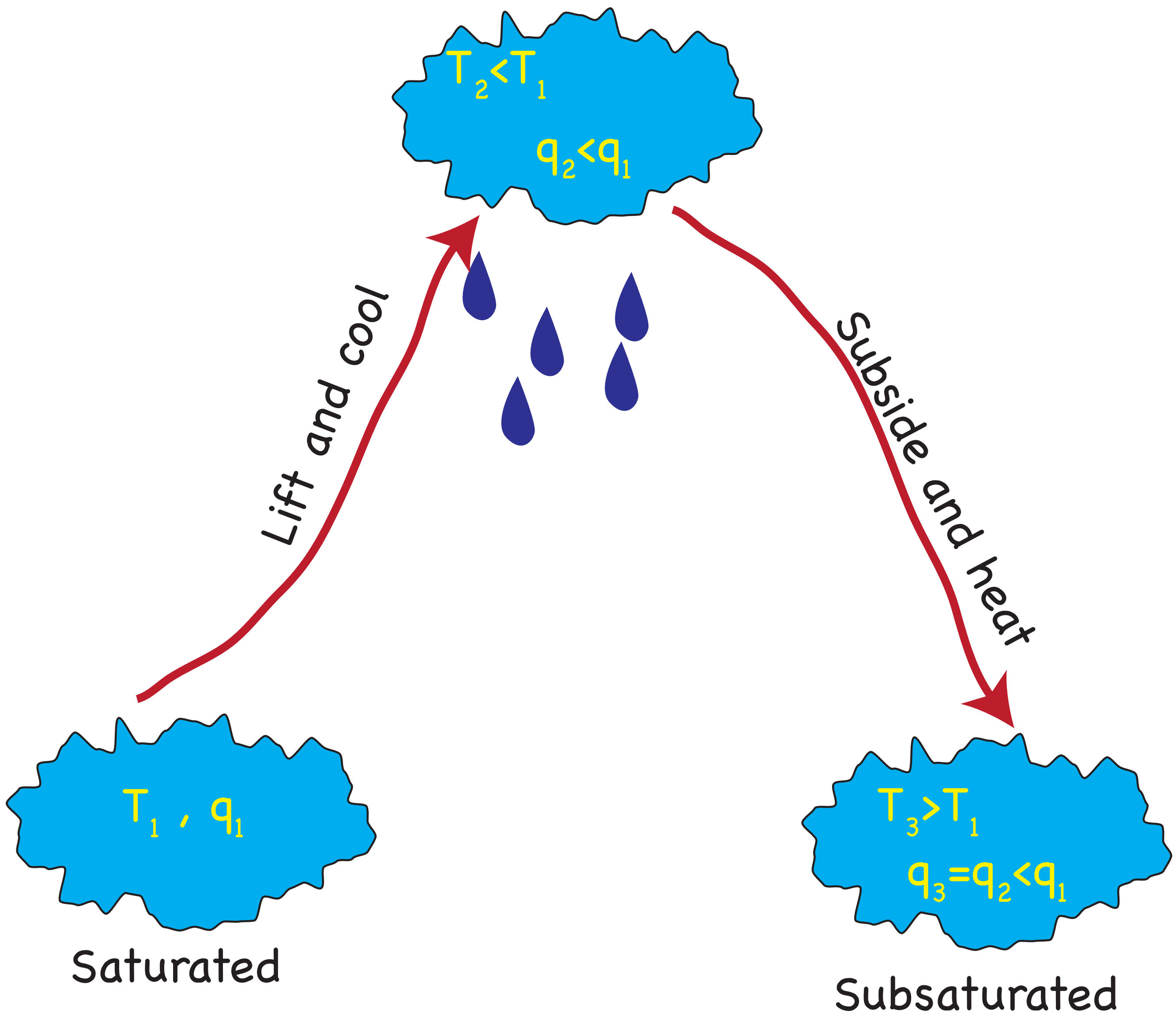


Synoptic eddies increase winter stability by sliding in cold polar air at low altitudes

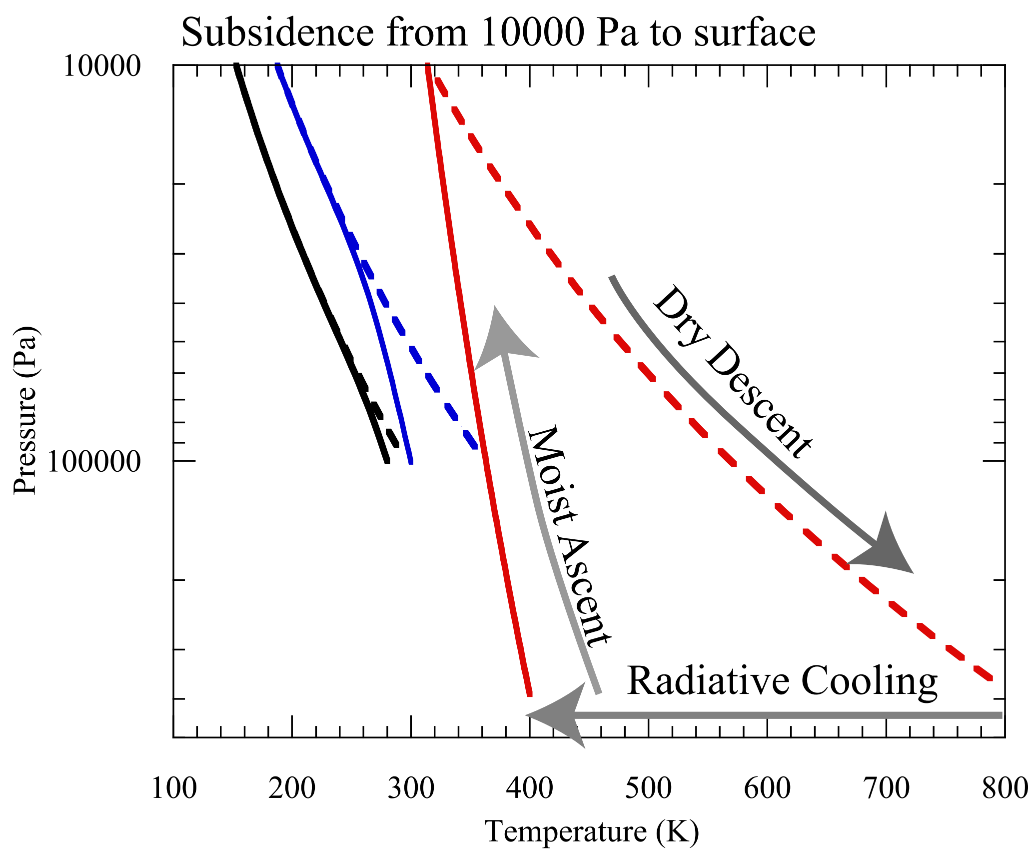
Beyond 1D: Subsaturation

- Atmospheres with a condensible component need not be saturated
- Subsaturation determines concentration of the condensible (e.g. water vapor)
- Subsaturation is a dynamical phenomenon
- Subsaturation affects runaway at both the inner and outer edge of the HZ.

Where does subsaturation come from?

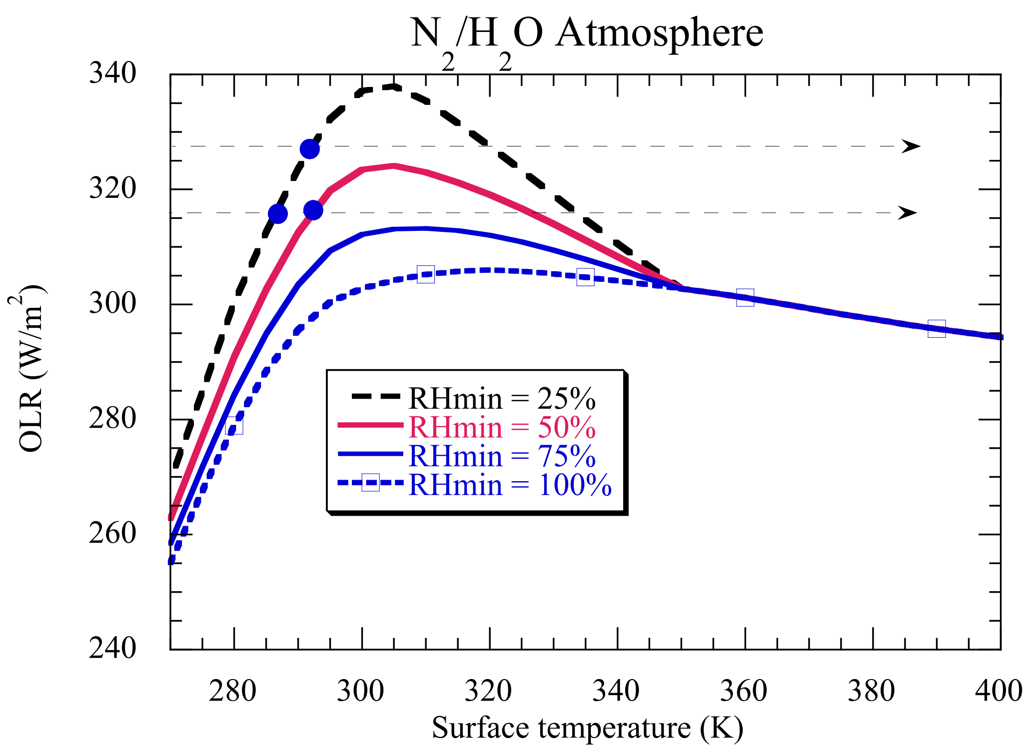


Rapid subsidence creates large T (and hence p) gradients



"Rapid" = "Rapid compared to radiative cooling"

RH feedback leads to metastable non-runaway states



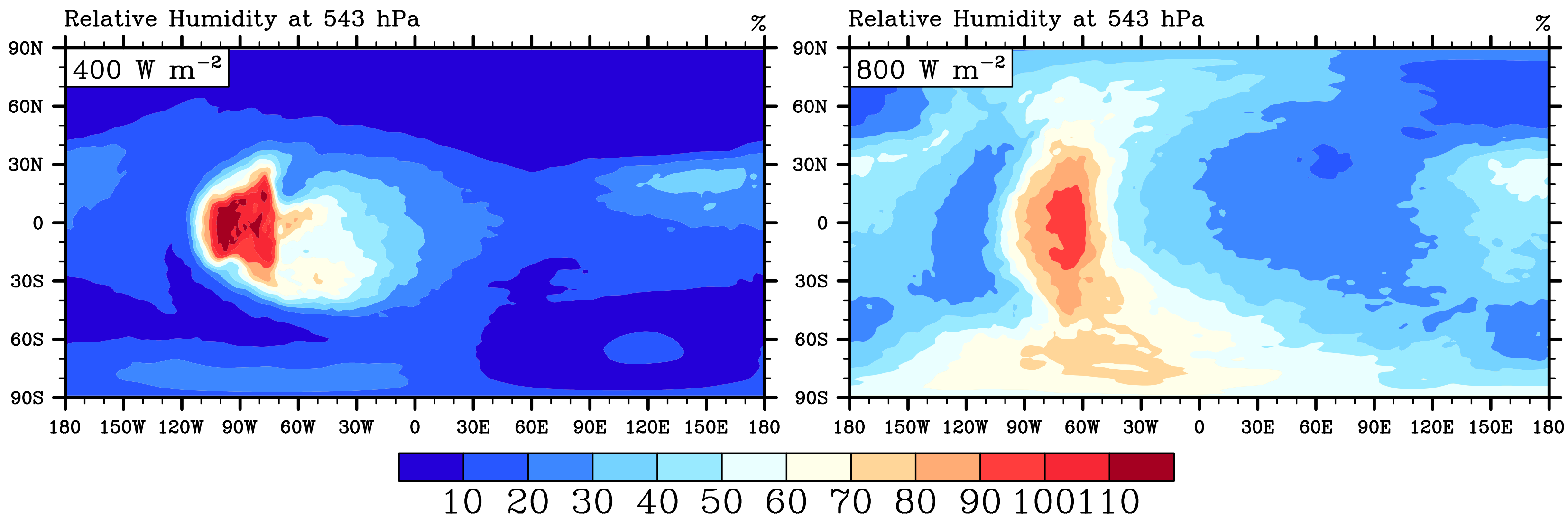
$$RH = RH_{min} + (100\% - RH_{min})(T - T_0)/(T_1 - T_0)$$

for $T_0 < T < T_1$

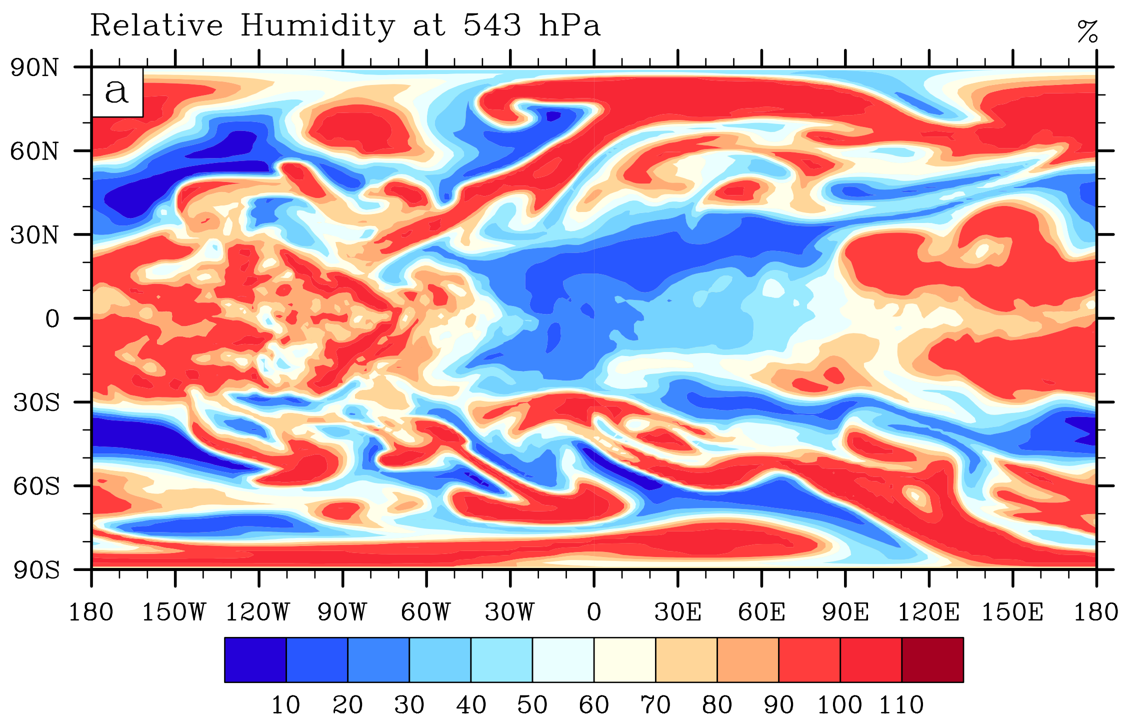
Subsaturation in FMS GCM dynamic simulations

- 3D dynamic general circulation model
- Tide-locked, various orbital periods
- Idealized moist thermodynamics (includes latent heat release)
- Grey gas radiation; no effect of condensible on IR optical depth
- Carried out by Feng Ding

Long period orbit



Short period orbit



A few take-home points

- Radiative-Convective modeling is still a valuable tool.
- Ideal for exploratory work on new problems, testing convection and radiation schemes, etc.
- Energy-conserving convective adjustment for atmospheres with condensation of a major constituent still has some wrinkles to be worked out.
- Even for planets with quite uniform surface temperature, dynamics has important zero-order climate effects via lapse rate, subsaturation and clouds.