Baroclinic Instability in Accretion Disks.
How Baroclinic are Protoplanetary Accretion Disks?

MAX-PLANCK-GESELLSCHAFT

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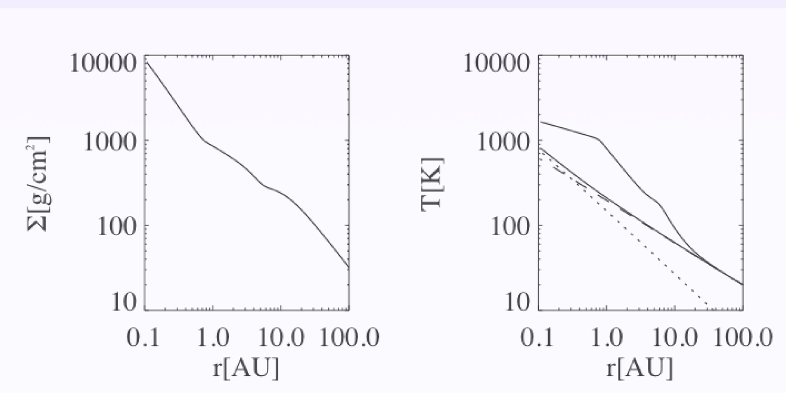
Introduction

Following recent debate on the nature of a radial entropy gradient driven instability in accretion disks and wether this instability falls under the same category as the Baroclinic Instability in planetary atmospheres, this paper discusses the similarities and differences of the Baroclinic state of the two different yet related systems. We show what the Baroclinicity of typical circumstellar disks is to expected, e.g. their thermodynamic structure plus thermal wind and vertical shear for typical disk parameters in agreement with observations. We also use these conditions to derive a typical growth rate for Baroclinic Instability following the Eady Model and find the same growth rate as for instability in Accretion Disks if neglecting the Keplerian shear. We therefor argue that the numerically found instability in circumstellar disks is indeed a special case of Baroclinic Instability for systems with strong radial (yet barotropic stable) shear.

Typical disk structure for a disk with $\dot{M} = 1E - 7M_{\rm sun}yr^{-1}$, a viscosity of $\alpha = 3E - 4$ around a pre-main sequence star with a $T_{\rm eff} = 4300K$, $M = 2M_{\rm sun}$ and $R = 2R_{\rm sun}$



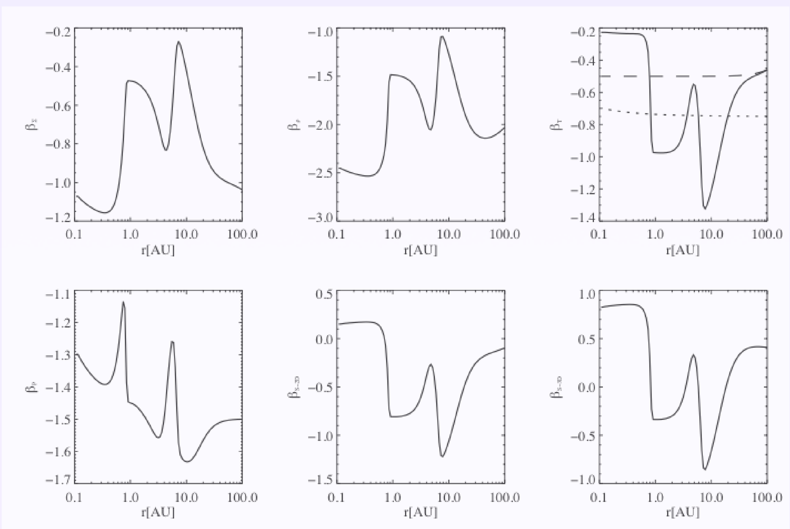


Figure 1: Surface density and temperature structure; Figure 2: Gradients in Surface density, density, temperature, pressure, and 2D/3D entropy: e.g. $\beta_\Sigma = -dlog\Sigma/dlogR$.

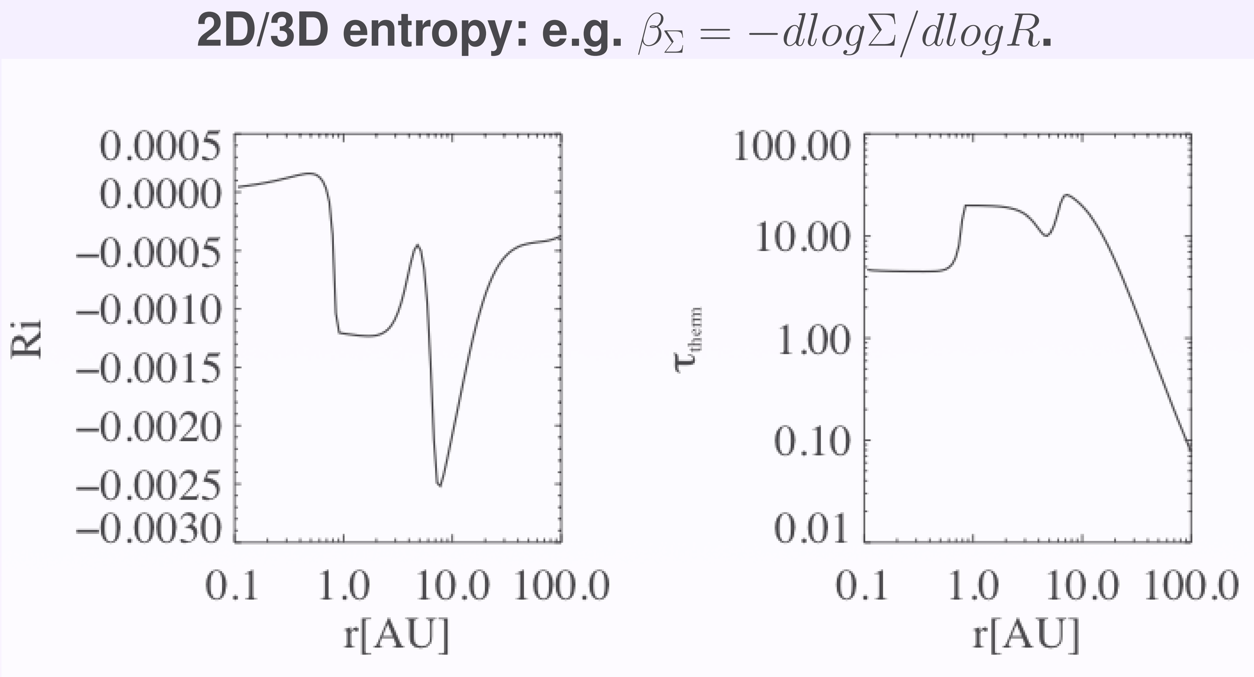


Figure 3: Rradial Richardson Number and cooling time. $Ri^2 = \frac{N^2}{(\frac{3}{2}\Omega)^2}$ with the radial Brunt-Väisälä frequency: $N^2 = -\frac{1}{\gamma}\left(\frac{H}{R}\right)^2\beta_s\beta_p\Omega^2$.

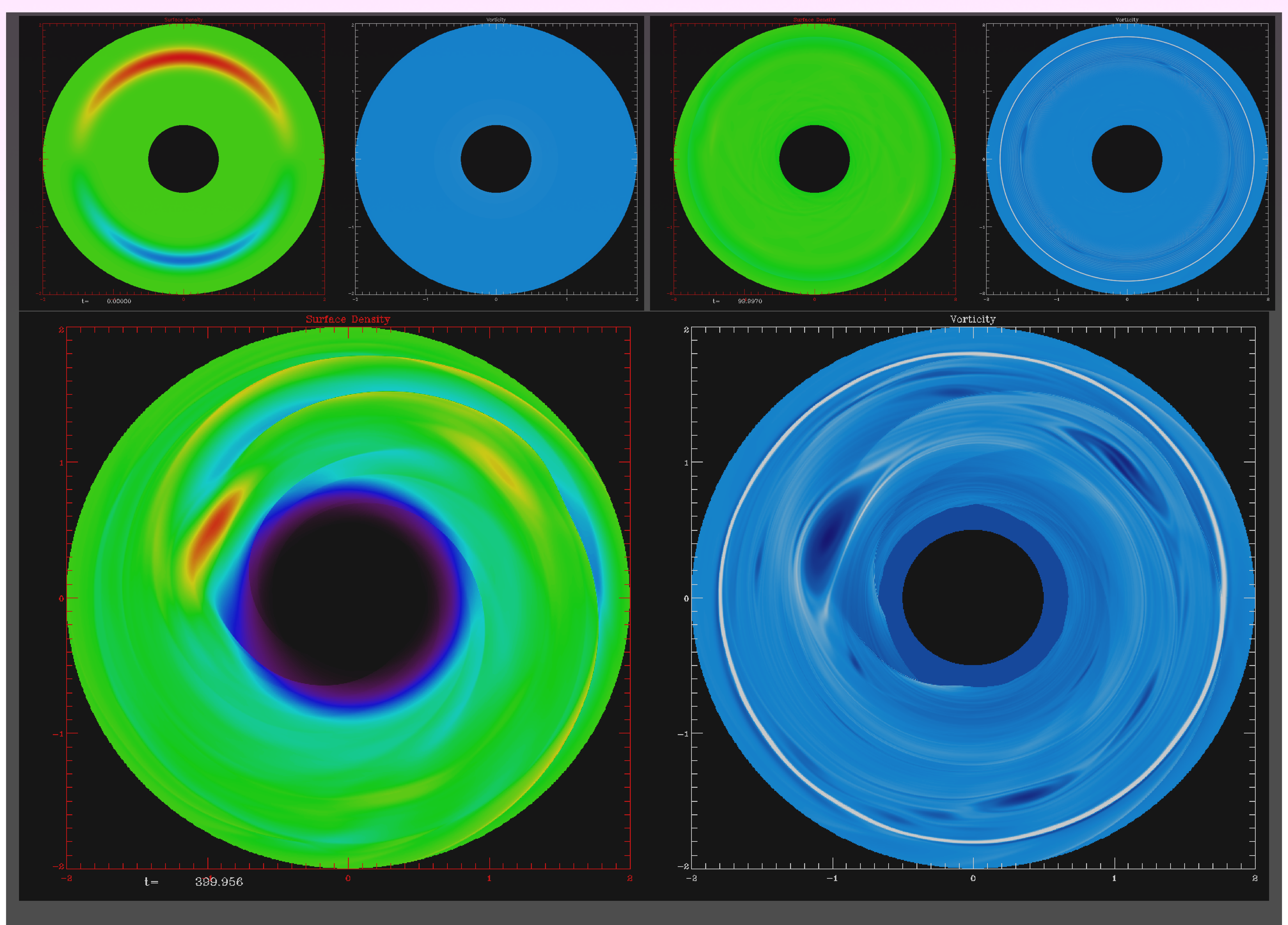


Figure 4: Pluto Code: Formation of Vortices.

Motivation

Klahr & Bodenheimer (2003) introduced the idea that a radial entropy gradient can generate turbulence in a Rayleigh and Solberg-Høiland stable disk and drive a significant angular momentum transport via Reynolds stresses. They had performed 3D radiation hydro simulations of global chunks of convection in accretion disks with vertical and radial entropy gradients and observed the formation of vortices and waves. They called this phenomenon the Global Baroclinic Instability (GBI), because it is the global radial entropy gradient which gives the baroclinic term in the hydro equations the possibility to obtain non-zero values for non-axissymetric entropy distributions.

Klahr (2004) performed a linear stability analysis for disks with a radial entropy gradient, yet found no linear stability. The obstacle to linear growth lies in the fact that the shear time is much shorter than the growth time and amplification is limited to the time when leading spiral waves turn into trailing ones. Lesur and Palaloizou (2010) and Lyra and Klahr (2011) find baroclinic generated vortices in 3D.

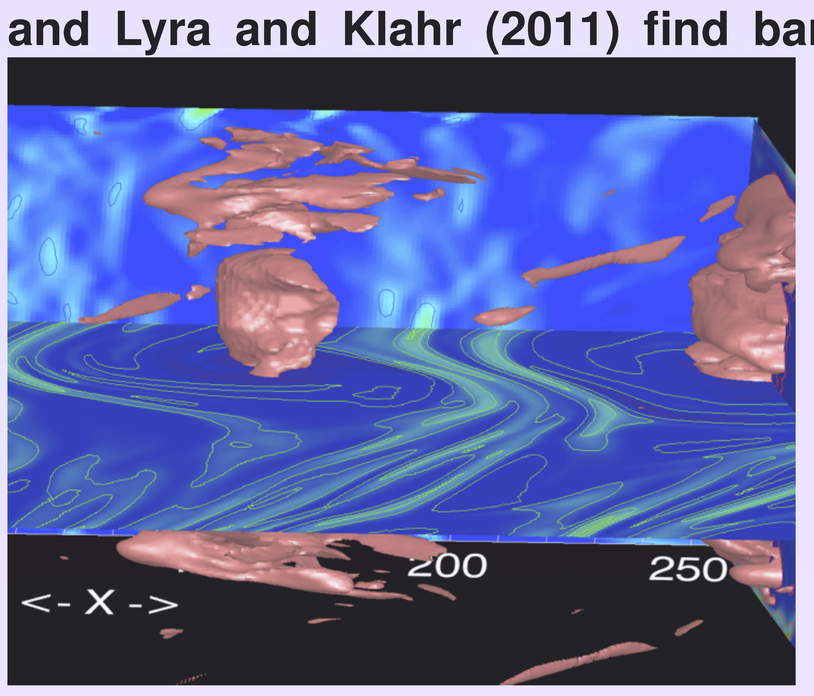


Figure 4: Tramp Code: Vortices in local stratified simulations.

Comparison with Planetary Baroclinic Instability

We use the Pluto Code for global models and the TRAMP and Pencil code for local simulations. The Global Models presented here are for vertically integrated density, but show the same behavior as the local 3D models which naturally include vertical density stratification and vertical shear following:

$$\Omega = \Omega_K \left[1 + \frac{1}{2}\left(\frac{H}{R}\right)^2 \left(\beta_\rho + \beta_T + \frac{\beta_T}{2}\left(\frac{z}{H}\right)^2\right)\right]. \quad (1)$$

We can define a typical vertical shear for the disk as $\delta v = U$ for $z = H$ and have

$$U = \frac{\beta_T}{4}\left(\frac{H}{R}\right)^2 \Omega R. \quad (2)$$

Plugging this in the equation for β we receive:

$$\beta = \frac{12}{\beta_T}\left(\frac{H}{R}\right)^{-3}. \quad (3)$$

For our standard values β obtaines values of 24.000.
Vertical Brunt-Väisälä Freq.:

$$N_z(H) = \sqrt{\frac{\gamma - 1}{\gamma}}\Omega \quad (4)$$

Deformation radius:

$$L_d = N_z H/\Omega = \sqrt{\frac{\gamma - 1}{\gamma}}H \quad (5)$$

Growth rate of B.I. in the Eady Model:

$$\sigma_e = U/L_D = \frac{\beta_T}{4}\sqrt{\frac{\gamma}{\gamma - 1}}\frac{H}{R}\Omega \quad (6)$$

which for $\beta_T = 0.5$ and $H/R = 0.1$ gives about 0.022Ω which is about 280 Orbits. Contrast this with the growth rate from Klahr (2004) for $n(t) = 0$ and $m > R/H$:

$$\Gamma_l = -im\frac{\frac{5}{4}\frac{\beta_T}{\gamma}}{m^2 + \frac{\kappa^2}{\Omega^2}\frac{R^2}{H^2}}\Omega \pm \frac{\beta_T}{\gamma}m\frac{H}{R}\Omega\frac{\sqrt{m^2 - \frac{33}{16}\frac{R^2}{H^2}}}{m^2 + \frac{\kappa^2}{\Omega^2}\frac{R^2}{H^2}} \quad (7)$$

$$= \frac{\beta_T}{\gamma}\frac{H}{R}\Omega. \quad (8)$$